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Energy-aware wireless networked control using radio-mode management

Carlos Canudas de Wit

CNRS-Gipsa-Lab, NeCS team, Grenoble, France

Madrid, May 2012

Material from:

- Energy-aware wireless networked control using radio-mode management,
  N. Cardoso de Castro, C. Canudas-de-Wit, and F. Garin. To appear in: ACC 2012, Montréal, Canada
- Smart Energy-Aware Sensors for Event-Based Control,
  N. Cardoso De Castro; D. E. Quevedo; F. Garin; C. Canudas-de-Wit. Sub. to: IEEE CDC'12

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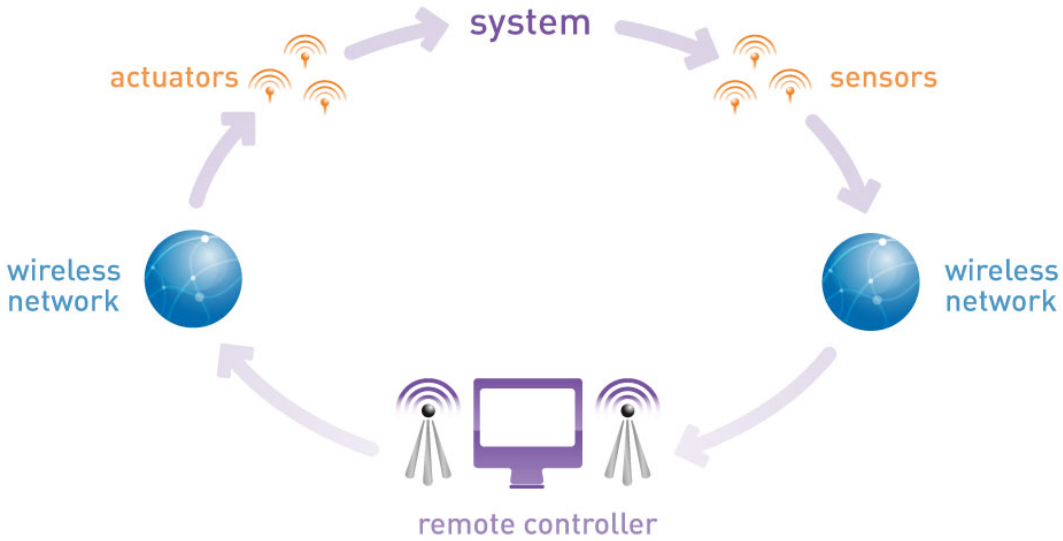
Context

Wireless Networked Control Systems



Challenges

- Limited energy resources
- Unreliable communications

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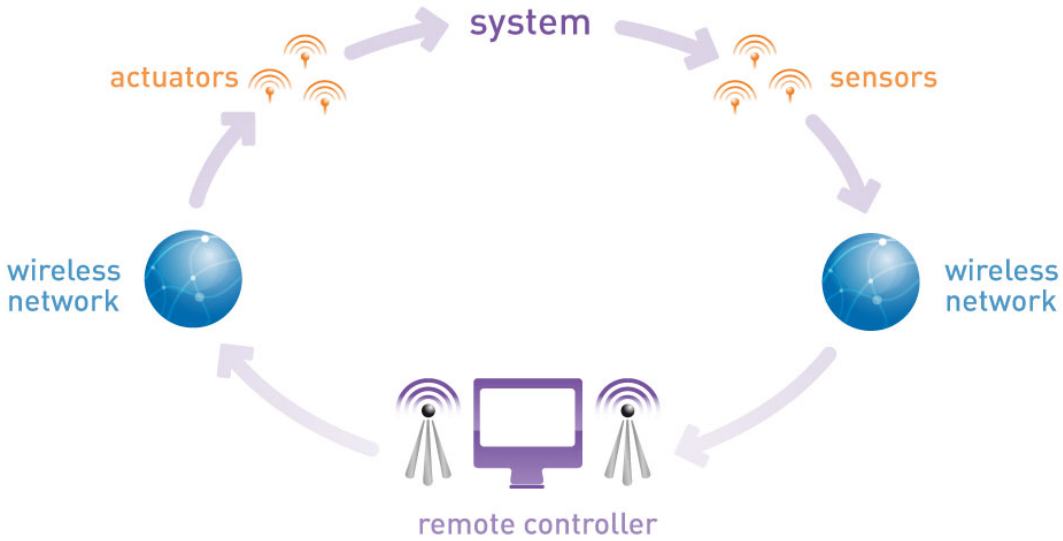

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Motivation

- The main energy consumption: in the radio chip,

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- The main energy consumption: in the radio chip,
- Classical approach: only ON/OFF modes,

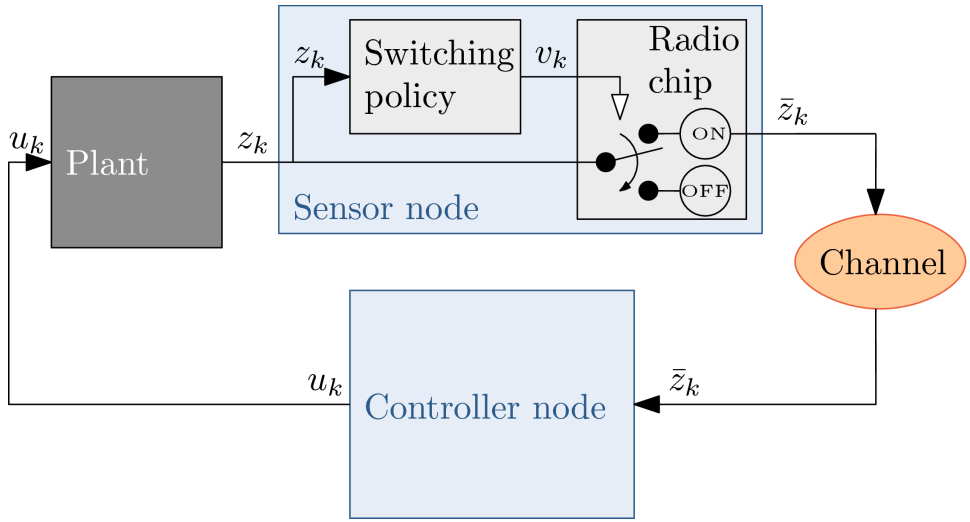

Figure: Control approach to limit communication.

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- The main energy consumption: in the radio chip,
- Classical approach: only ON/OFF modes,
- Novelty here: multi radio-modes (i.e. ON, IDLE1, IDLE2,.. OFF), with energy transition weights

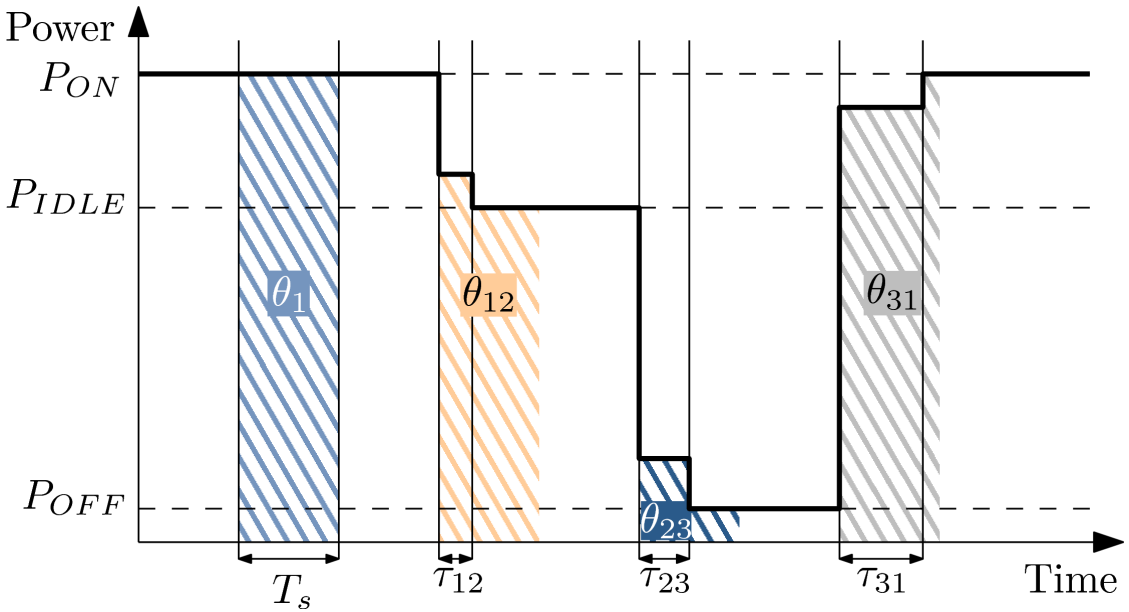

Figure: Temporal use of 3 radio-modes (ON, IDLE, OFF).

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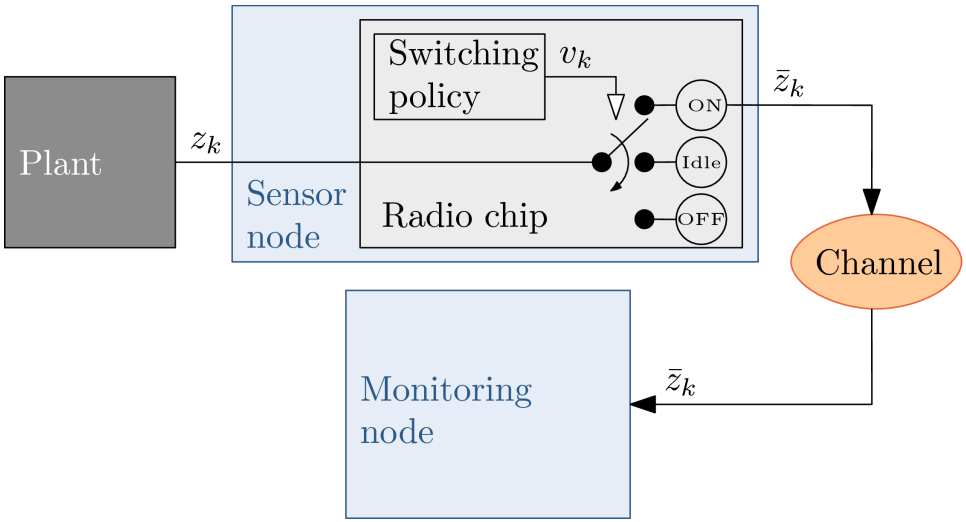

Figure: Communication approach to save energy.

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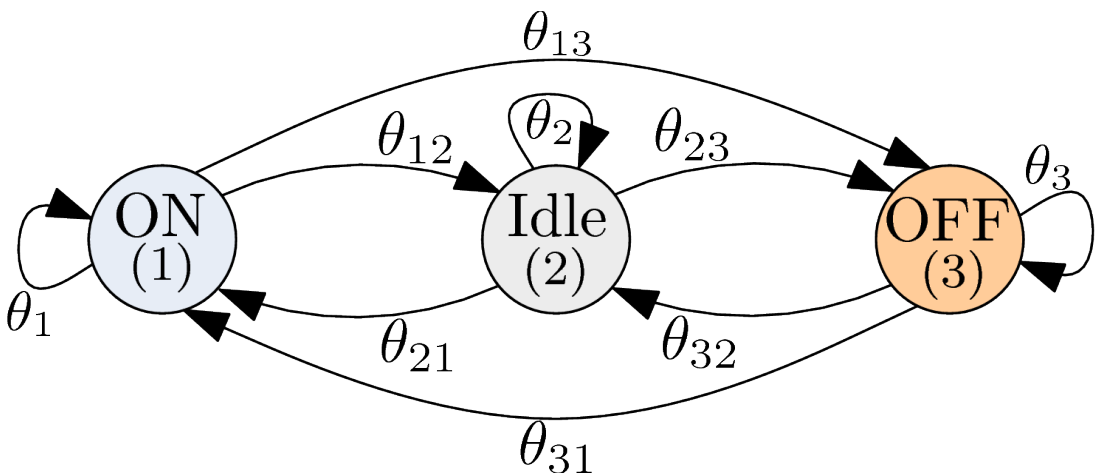
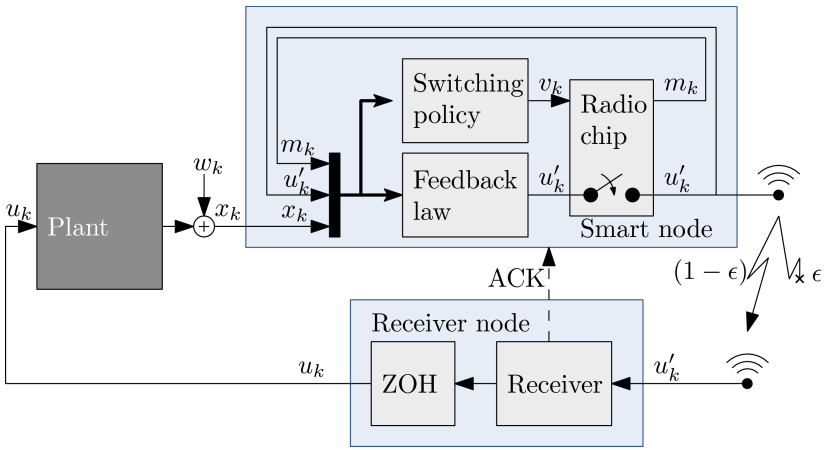
Objective

- Multiple radio-modes (e.g., ON, IDLE, OFF)
- Explicit radio-mode (scheduling) law depending from plant's state
- Control and communication co-design: feedback K and radio-mode law
- Optimize the Trade-off: quality of control vs. energy used for transmission (sensor life-time)



Problem setup

Discrete-time unstable linear plant $z_{k+1} = Az_k + Bu_k$
with (A, B) controllable
Goal: stabilize z to zero.



- Sensor node:
  - measures z_k
  - knows u_{k-1}
  - knows current radio mode m_k

  It computes:
  - next radio-mode m_{k+1}
  - $\tilde{u}_k = -Kz_k$ to be transmitted to the actuator if $m_{k+1} = \text{ON}$
- Actuator node applies

$$u_k = \begin{cases} \tilde{u}_k & \text{if received} \\ u_{k-1} & \text{otherwise} \end{cases}$$

Trade-off LQ cost vs energy

Goal:

policy for sensor's radio-mode, optimal trade-off LQ vs energy

- At k-th sampling period:
  - $\theta_{m_k, m_{k+1}}$ = energy spent in transition from mode m_k to m_{k+1}
  - LQ cost describes the quality of the stabilizing control
- Natural definition of cost-to-go (cost in one time interval):

$$\ell_{m_{k+1}}(z_k, u_{k-1}, m_k) = z_k^T Q z_k + u_k^T R u_k + \theta_{m_k, m_{k+1}}$$

- Would like to optimize a cost such as

$$J = \sum_{k=0}^{\infty} \ell_{m_{k+1}}(z_k, u_{k-1}, m_k)$$

but the above formula is meaningless:
$J = \infty$ for any choice of the modes $\{m_k\}_{k \in \mathbb{N}}$.

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Trade-off LQ cost vs energy (2)

We consider two costs:

Infinite time-horizon, with discount factor $\lambda \in (0, 1)$:

$$J_{\{m_k\}}(z_0, u_{-1}, m_0) = \sum_{k=0}^{\infty} \lambda^k \ell_{m_{k+1}}(z_k, u_{k-1}, m_k)$$

Finite and receding time-horizon (model-predictive control):

at time k, optimize over horizon $[k+1, \ldots, k+H]$

$$J_{m_{k+1},\ldots,m_{k+H}}(z_k, u_{k-1}, m_k) = z_{k+H}^T Q_f z_{k+H} + \sum_{t=k}^{k+H-1} \ell_{m_{t+1}}(z_t, u_{t-1}, m_t)$$

then apply only m_{k+1}^*, and at time $k+1$ consider horizon $[k+2, \ldots, k+H+1]$, i.e., optimize $J_{m_{k+2},\ldots,m_{k+H+1}}(z_{k+1}, u_k, m_{k+1})$.

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Infinite horizon – overview

- Problem: find sequence $m_1, m_2, \ldots$ minimizing

$$J_{\{m_k\}}(z_0, u_{-1}, m_0) = \sum_{k=0}^{\infty} \lambda^k \ell_{m_{k+1}}(z_k, u_{k-1}, m_k)$$

- Dynamic programming (value iteration method)
  - gives algorithm to solve the problem;
  - ensures that the optimal solution is stationary, i.e., m_{k+1}^* depends only on state z_k, u_{k-1}, m_k, not on time index k
    $\Rightarrow$ optimization algorithm run off-line; optimal solution applied off-line (e.g. look-up table)
- Good news: nice simulation results
  (trajectories driven near 0, with low energy consumption)
- Bad news: because of discount factor λ, J^* can't be used as a Lyapunov function to prove stability.

Value Iteration Method

Value iteration:

- initialize $V_0 \equiv 0$, and then iterate

$$V_{i+1}(z,u,m) = \min_{\mu \text{ mode}} \left[\lambda V_i(z^+(\mu), u^+(\mu), \mu) + \ell_\mu(z,u,m)\right]$$

Good properties:

- Value iteration converges to optimal cost function:

$$\lim_{i\to\infty} V_i(z_0, u_{-1}, m_0) = \min_{\{m_k\}} J_{\{m_k\}}(z_0, u_{-1}, m_0)$$

- arg min gives a stationary policy, i.e., choice of mode depending only on (z,u,m), not on time index k

$$\mu^*(z,u,m) = \lim_{i\to\infty} \arg\min_{\mu \text{ mode}} \left[\lambda V_i(z^+(\mu), u^+(\mu), \mu) + \ell_\mu(z,u,m)\right]$$

Value Iteration Method (2)

- Difficulty: in general, value iteration method requires to compute functions of $V_i(z, u, m)$, which can be approximated by discretizing the space state + interpolation
- For some problems, can do better: recognize that the Value Function V_i has a given shape and can be parametrized easily (i.e. LQ control)
- In our problem:

$$V_i(z, u, m) = \min_{(\Pi, \pi) \in \mathcal{P}_i} \left[(z^T, u^T) \Pi \begin{pmatrix} z \\ u \end{pmatrix} + \pi_m \right]$$

  - at each i-th iteration compute a set $\mathcal{P}_i$ of pairs (Π, π), with Π square matrix of size (size(z)+size(u)), π a vector of size = number of modes, and π_m an element of π.
  - card($\mathcal{P}_i$) is finite, but grows exponentially with i

Simulation results (off-line optim.)

Simple example

(scalar z_k and u_k; 3 modes)

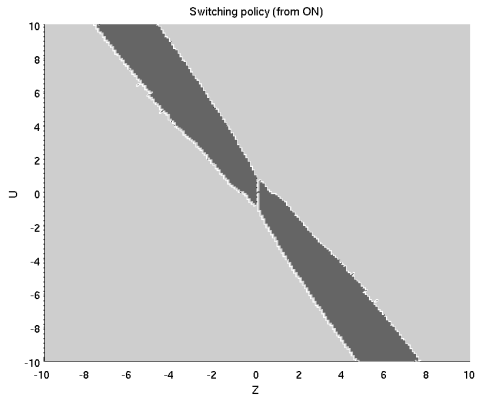
$z_{k+1} = 2z_k + u_k, \quad u_k = -1.6z_k,$

$Q = 1,\ R = 0.5\ ,\ \lambda = 0.8,$

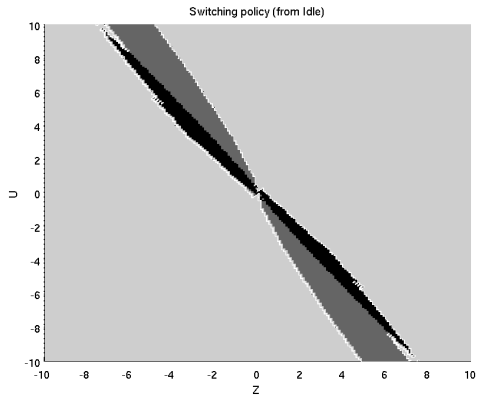
$\theta = \begin{bmatrix} 10 & 4 & 7 \\ 4 & 2 & 1 \\ 7 & 1 & 0 \end{bmatrix}$

Color $= m_{k+1}$

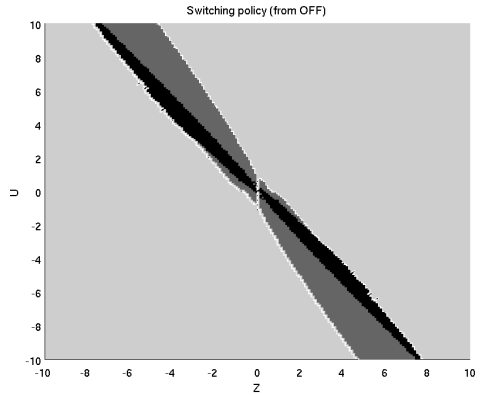
(black $=$ OFF; lightest gray $=$ ON)



Current mode $m_k =$ ON



Current mode $m_k =$ IDLE



Current mode $m_k =$ OFF

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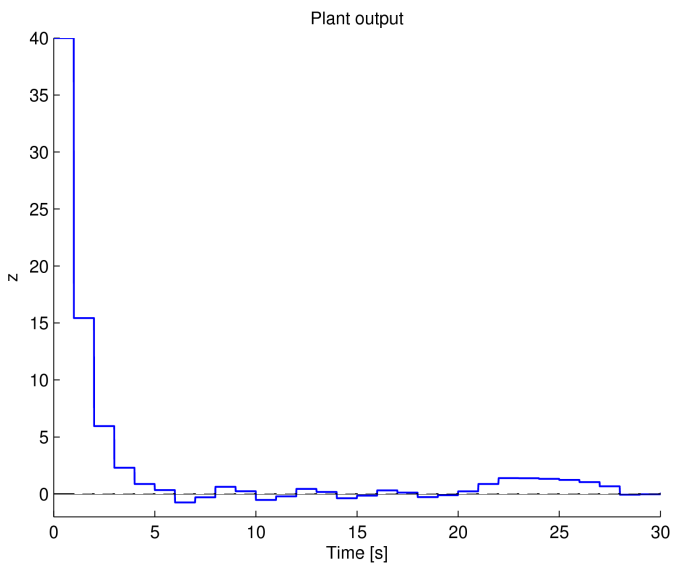
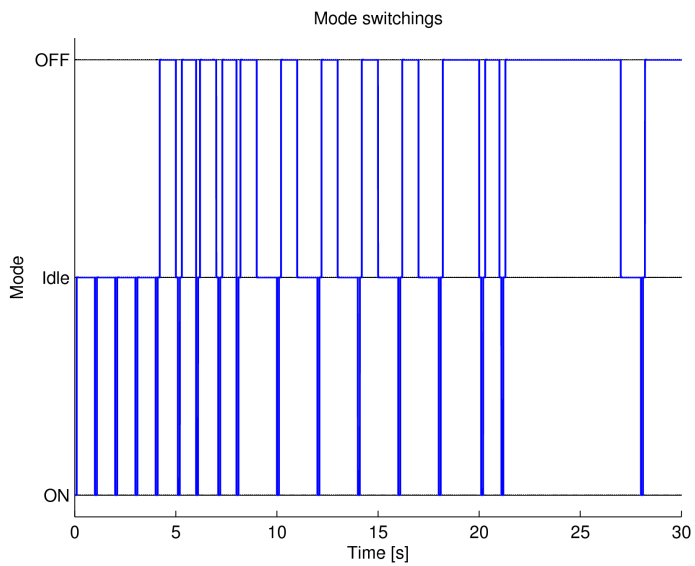
Simulation results (trajectories)

Same example
Trajectories of state z_k and mode m_k
obtained applying the policy computed off-line



State trajectory



Mode trajectory

Finite horizon (Model-Predict. Contr.)

Submitted CDC2012, in col. with Daniel Quevedo, Univ. Newcastle

- Optimization over a receding time-horizon of length H:
  at time k, optimize over horizon $[k, \dots, k+H]$

$$J_{m_{k+1},\dots,m_{k+H}}(z_k, u_{k-1}, m_k) = z_{k+H}^T Q_f z_{k+H} + \sum_{t=k}^{k+H-1} \ell_{m_{t+1}}(z_t, u_{t-1}, m_t)$$

  and apply only first step m_{k+1}^* of optimal vector $m_{k+1}^*, \dots, m_{k+H}^*$
- Optimization solved with dynamic programming:
  initialize $J_0^*(z, u, k) = z^T Q_f z$, then for $i = 0, \dots, H-1$

$$J_{i+1}^*(z, u, k) = \min_{\mu \text{ mode}} \left[J_i^*(z^+(\mu), u^+(\mu), \mu) + \ell_\mu(z, u, m) \right]$$

  $\Rightarrow J_H^*(z_k, u_{k-1}, m_k) = \min_{m_{k+1},\dots,m_{k+H}} J_{m_{k+1},\dots,m_{k+H}}(z_k, u_{k-1}, m_k)$.
- Similarly to infinite-horizon:
  - solve dyn. progr. off-line, apply optimal policy on-line
  - can exploit the shape of the value function, compute a set $\mathcal{P}_i$ of pairs (matrix, vector)

Stability of the MPC solution

- With suitable choice of Q_f can prove practical stability, i.e.: $\exists$ invariant and globally attractive neighborhood of 0
- We have proof for noiseless system, but hope to extend it
- Proof uses Lyapunov fct $V(z_k, u_{k-1}, m_k) = J_H^*(z_k, u_{k-1}, m_k)$, i.e., optimal cost over the time window $[k+1, \ldots, k+H]$
  - V is a Lyapunov function in the ISpS sense (Input-to-State practical Stability), i.e.:
    - $\exists$ $\mathcal{K}_\infty$ fcts α_1, α_2 and constant $c_1 \geq 0$ s.t.
    $$\alpha_1(\|z\|) \leq V(z, u, m) \leq \alpha_2(\|z\|) + c_1$$
    - $\exists$ $\mathcal{K}_\infty$ fct α_3 and constant $c_2 \geq 0$ s.t.
    $$\Delta V(z, u, m) \leq -\alpha_3(\|z\|) + c_2$$
    - a fct $\alpha : [0, +\infty) \to [0, +\infty)$ is $\mathcal{K}_\infty$ if $\alpha(0) = 0$, α is continuous, increasing and $\lim_{x \to \infty} \alpha(x) = +\infty$
  - existence of ISpS Lyapunov fct. implies practical stability see [Raimondo et al, Min-max MPC of nonlin. systems: a unifying overview on stability, Europ. J. Contr. 2009]

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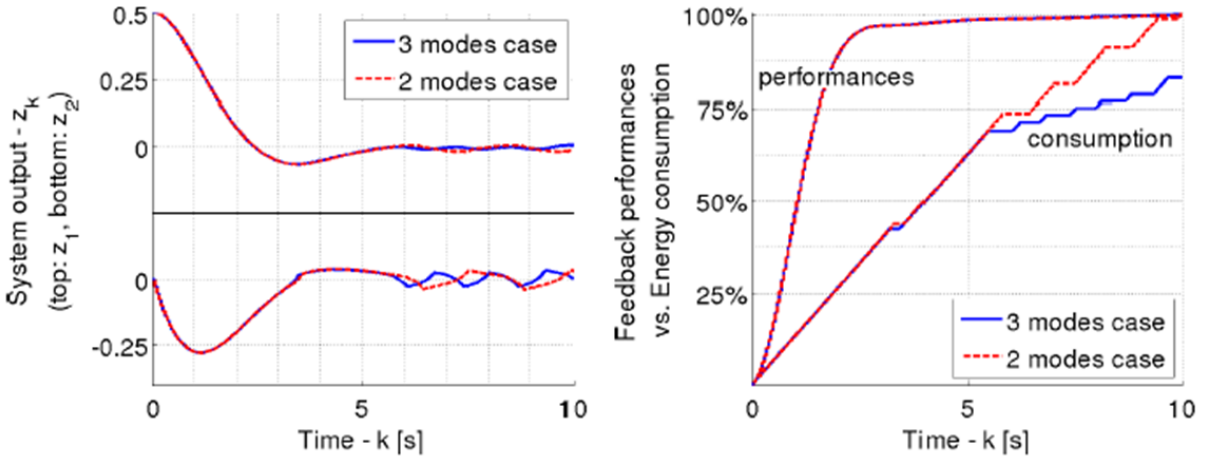
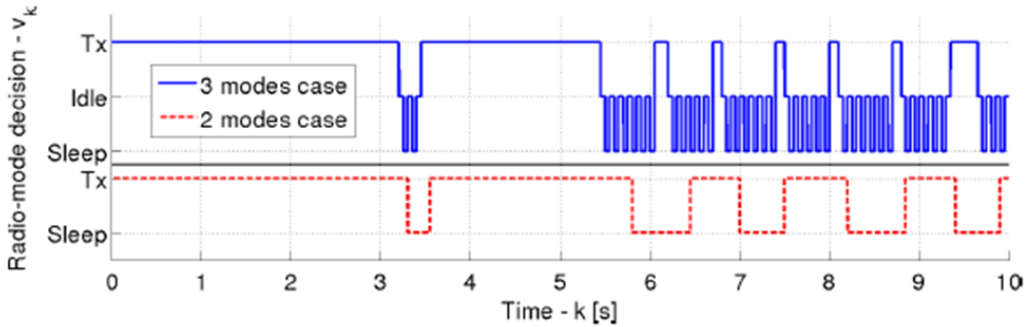
Simulation results MPC–approach



(a) Output of the system x_k. The first component of each case is plotted in the top and the second in the bottom of the figure.

(b) Feedback performances $\sum_{i=0}^{k}(x_i^\top \bar{Q} x_i + u_i^\top \bar{R} u_i)$ (upper curve) and the cumulated energy consumption E_k. Curves are normalized to the largest value at the end of the simulation.



(c) Switching decision v_k.

Fig. 3. On-line simulations comparing the case with 2 and 3 radio modes. The 3 modes case in plotted in blue and the 2 modes case in dashed red.

Figure: Simulation with multi-mode using MPC .

Variations on the theme

Other work done or in progress, both on infinite horizon and MPC:

- Optimize control law: $u_k =$ any fct of z_k
  instead of a feedback $u_k = -Kz_k$ for a given fixed K
  (similar to LQ)
- Packet drop in the channel:
  - memoryless packet-erasure channel with acknowledgements
  - optimize average performance
  - we have nice simulations
    (pract. stability preserved with erasure prob $= 30\%$)
  - TBD: multiple transmitting radio-modes, with different power $\leftrightarrow$ different erasure probabilities
- Noisy plant (or measurements):
  - simulations ok
  - TBD: accomodate noise in the MPC stability proof

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Conclusion

- We have considered the radio-mode policy of wireless a sensor on a simple networked control loop
- We have found the optimal policy for some costs describing a trade-off stabilization vs low-energy,
  - on an infinite time-horizon (with a discount factor)
    see ACC'12 Cardoso, Canudas, Garin
  - on a finite receding time-horizon (MPC)
    submitted to CDC 2012, collaboration with Daniel Quevedo
- Future research:
  - hardware simulations (at inria's SenseLab)
  - network of multiple sensors and actuators