Sistemas de control reseteado

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1. Motivación y conceptos básicos (reset)
2. Estabilidad de sistemas de control reseteado
3. El controlador PI+CI
4. Banda de reset/reset variable
5. Métodos de ajuste de p_{reset}

Conclusiones

El integrador de Clegg (1958)

"A nonlinear integrator for servomechanisms"

- **Idea básica**: el estado (la salida) del sistema se pone a cero (reseteo) en los instantes en que la entrada es cero.

$$\begin{cases} \dfrac{dy(t)}{dt} = u(t),\ u(t) \neq 0 \\ y(t^+) = 0,\ u(t) = 0 \end{cases}$$

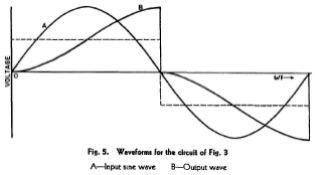
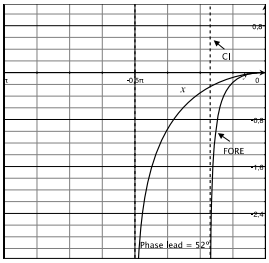


Fig. 5. Waveforms for the circuit of Fig. 3
A—Input sine wave B—Output wave

- CI proporciona fase extra en relación a un integrator (lineal)

$$CI(\omega) = \frac{1.62}{\omega} e^{-j38.1^\circ}, \quad I(j\omega) = \frac{1}{\omega} e^{-j90^\circ}$$



Horowitz's FORE (1975)

- FORE gives phase lead over a (linear) first order system:

$$\begin{cases} \dfrac{dy(t)}{dt} = -ay(t) + Ku(t),\ u(t) \neq 0 \\ y(t^+) = 0,\ u(t) = 0 \end{cases}$$

$$FORE(j\omega) = \frac{K}{a + j\omega}\left(1 + j\frac{2}{\pi}\frac{\omega^2}{a^2 + \omega^2}(1 + e^{-a\frac{\pi}{\omega}})\right)$$

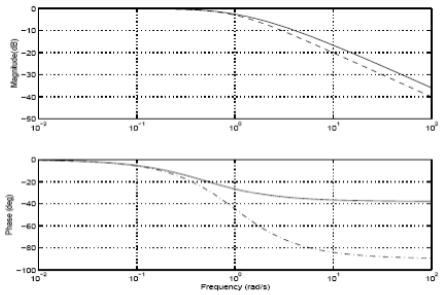
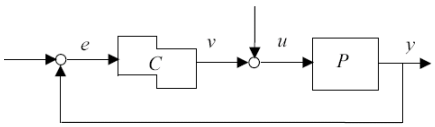


Fig. 4. Bode Diagram of FORE DF (solid line) and its LTI base system (dashed line)

Formulación básica



$$C: \begin{cases} \dot{x}_r(t) &= A_r x_r(t) + B_r e(t), & e(t) \neq 0 \\ x_r(t^+) &= A_\rho x_r(t), & e(t) = 0 \\ v(t) &= C_r x_r(t) + D_r e(t) \end{cases}$$

$$P: \begin{cases} \dot{x}_p &= A_p x_p + B_p u \\ y &= C_p x_p \end{cases}$$

- El sistema base es LIT
- Los instantes de reseteo se definen en los cruces de la señal de error con cero
- Los eventos de reseteo son acciones **dependientes** del estado del sistema de control

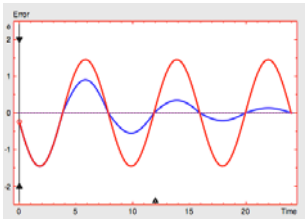
- Sistema en bucle cerrado (no forzado): **ecuación diferencial lineal impulsiva** (impulsos dependientes del estado)

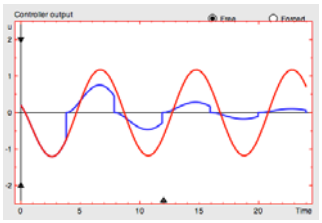
$$\begin{cases} \dot{x}(t) &= Ax(t), & x \notin \mathcal{M} \\ x(t^+) &= A_R x(t), & x \in \mathcal{M}, \end{cases} \qquad A = \begin{pmatrix} A_p & B_p C_r \\ -B_r C_p & A_r \end{pmatrix}$$

$$A_R = \operatorname{diag}(I_{n_p}, A_\rho) = \operatorname{diag}(I_{n_p}, (I_{n_p}, O_{n_\rho}))$$

- La superficie de reset *M* define los instantes de reseteo $M = \{x \in R^n : Cx = 0\}, \; C = \begin{bmatrix} C_p & 0 \end{bmatrix}$
- La superficie "after-reset" M_R, $M_R = A_R M = \{y \in R^n : y = A_R x, \; Cx = 0\}$

Solución del sistema de control ("well-posedness")

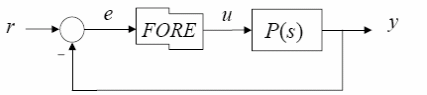




$$t_k = \tau_k(x_0), \; k = 0,1,2,\ldots \qquad \begin{cases} x(t) = e^{A(t-t_k)} A_R e^{A(t_k - t_{k-1})} \ldots A_R e^{A(t_1 - t_0)} x_0, & t \in (t_k, t_{k+1}) \\ x(t_k^+) = A_R x(t_k) \end{cases}$$

- La solución está bien definida si (para cualquier condición inicial admisible)
  - Los instantes de reseteo están bien definidos y son distintos
  - Existe un número finito de reseteos en cualquier intervalo acotado
- Una método simple de evitar problemas (bloqueos, soluciones tipo Zenón) es usar regularización temporal: $t_{k+1} - t_k \geq \rho > 0$
- Una condición geométrica: Los estados "after-reset" son observables

Ejemplo

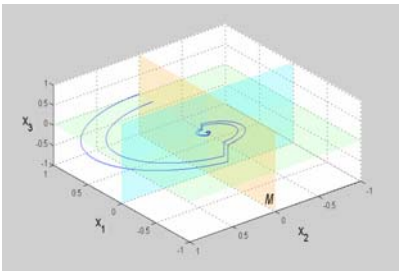


$$P(s) = \frac{s+1}{s(s+0.2)}$$

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & -0.2 & 1 \\ 0 & -1 & -1 \end{pmatrix}, A_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$



$M = \{x \in \mathbb{R}^3 | \; x_2 = 0\}$, $M_R = \{x \in \mathbb{R}^3 | \; x_2 = x_3 = 0\}$

Condición de "well-posedness" : El subespacio de estados no observables es

$$\mathcal{N}\begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix} = \mathcal{N}\begin{pmatrix} 0 & 1 & 0 \\ 1 & -0.2 & 1 \\ -0.2 & -0.96 & -0.2 \end{pmatrix} = span\{(1,0,-1)^T\}$$

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Estabilidad (Lyapunov)

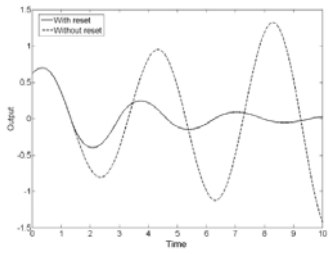
$$\begin{cases} \dfrac{dx(t)}{dt} = Ax(t),\ x(t) \notin M \\ x(t^+) = Bx(t),\ x(t) \in M \end{cases}$$

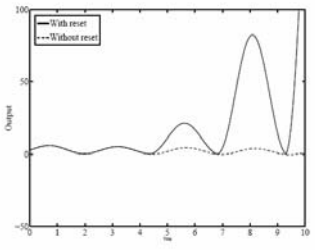
- El sistema es estable si para cualquier $\varepsilon > 0$ existe un $\delta > 0$ tal que para cualquier condición inicial x_0 que cumpla $||x_0|| < \delta$, se tiene $||x(t)|| < \varepsilon$ para $t>0$.
- El sistema es atractivo si $x(t) \to 0$ para $t \to \infty$
- El sistema es asintóticamente estable si es estable y atractivo.

- El sistema base (flujo) es asintóticamente estable si y sólo si ***A*** es Hurwitz
- El sistema discreto (salto) es asintóticamente estable si y sólo si ***B*** es Schur
- **El sistema reseteado es asintóticamente estable si y sólo si ???**

Estabilidad (Lyapunov)

El problema no es trivial: las acciones de reseteo pueden estabilizar un sistema base inestable,
... pero pueden también desestabilizar un sistema base estable

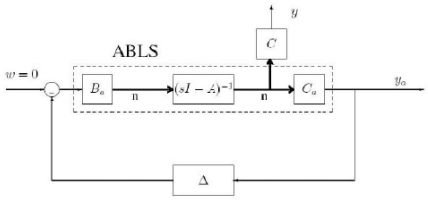




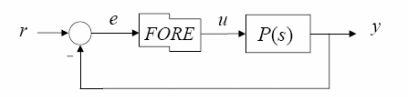
Resultados independientes de los instantes de reset: ***H_β*-condition**

Resultados dependientes de los instantes de reset: condiciones basadas en un “dwell-time”.

Resultados indepedientes de los instantes de reset: condición H_β

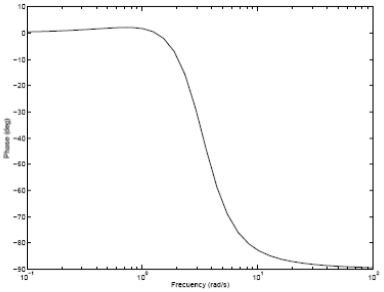


Ejemplo



$$A_p = \begin{bmatrix} -6 & -4 \\ 2 & 0 \end{bmatrix} \quad B_p = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad C_p = \begin{bmatrix} 0.5 & 0.75 \end{bmatrix} \quad D_p = 0$$

$$H_\beta = \frac{s^2 + 6.102s + 8.306}{s^3 + 7s^2 + 23s + 35}$$

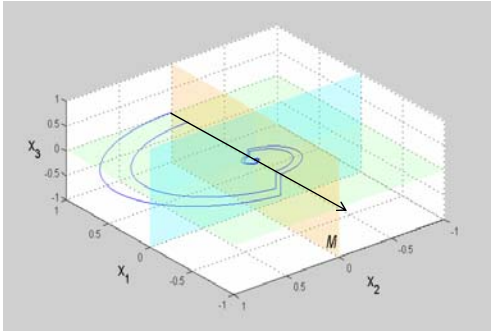


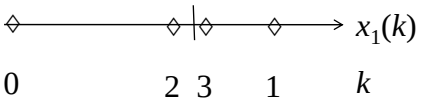
Resultados dependientes de los instantes de reset: "dwell-time"

Stability problem reduces to check the stability of the LTV discrete system

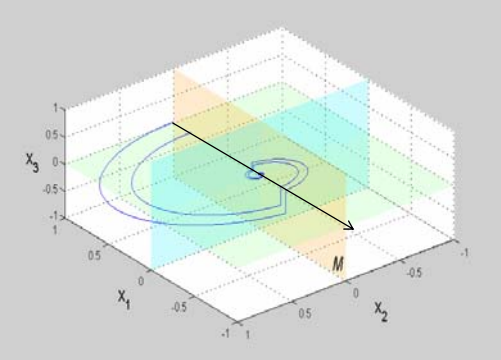
$$x_r(k+1) = A_R e^{A\Delta_k} x_r(k)$$

$$\begin{cases} \dfrac{dx(t)}{dt} = Ax(t),\ x(t) \notin M \\ x(t^+) = A_R x(t),\ x(t) \in M \end{cases}$$





- For low order systems reset is periodic, but this is not true in general.
- In these simple cases, with constant reset interval Δ, stability problem reduces to check if the matrix $A_R e^{A\Delta}$ is Schur



Example

Δ = 3.16

$x_1(k+1) = -0.30x_1(k)$

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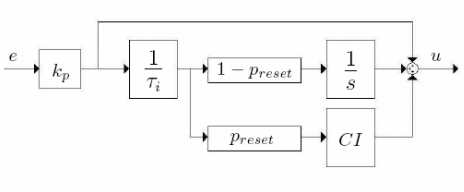
3. El controlador PI+CI



$$C:\begin{cases}\dot{x}_r(t) &= A_r x_r(t)+B_r e(t),\ e(t)\neq 0\\ x_r(t^+) &= A_\rho x_r(t), \qquad e(t)=0\\ v(t) &= C_r x_r(t)+D_r e(t)\end{cases}$$

$$A_r=\begin{pmatrix}0&0\\0&0\end{pmatrix}\quad C_r=\frac{k_p}{\tau_i}\begin{pmatrix}1-p_{reset}\ p_{reset}\end{pmatrix}$$

$$B_r=\begin{pmatrix}1\\1\end{pmatrix}\quad D_r=k_p\quad A_\rho=\begin{pmatrix}1&0\\0&0\end{pmatrix}$$

- A simple structure easily implementable, with few parameters
- Application target: process control
- Hopefully good transitory and steady state properties
- Also antiwindup behavior
- "Simple" tuning rules
  - Tune the base PI compensator
  - Select the reset percentage to reduce overshoot
- A fast response with no excessive overshoot may be obtained, overcoming LTI compensation limitations.
- Very intuitive for manual tuning: reset appears as a single parameter p_{reset}
- CI: A derivative action without increasing cost of feedback

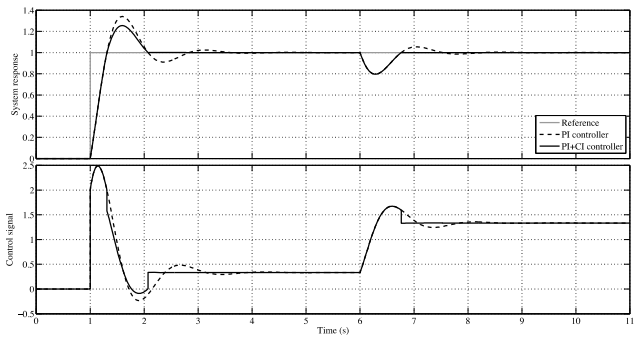
Sistemas de primer orden (FO)

$$\alpha=\frac{1+k_p k}{2\tau}$$

$$\beta=+\sqrt{\left|\alpha^2-\frac{k_p k}{\tau\tau_i}\right|}$$

1. Design the PI controller to obtain k_p and τ_i, for a desired oscillatory system response ($\left(\frac{1+kk_p}{2\tau}\right)^2 < \frac{kk_p}{\tau\tau_i}$) by using any tuning rules.
2. Compute α and β by using (4.7) and (4.8).
3. Finally, calculate the reset ratio, p_{reset}, by using (4.14).

$$p_{reset}=\frac{e^{-\frac{\alpha\pi}{\beta}}}{1+e^{-\frac{\alpha\pi}{\beta}}}$$



Sistemas de primer orden con retardo (FOPDT)

$$P(s) = \frac{k}{\tau s + 1} e^{-hs}$$

Ajuste del PI base mediante IMC/SIMC

$$k_p = \frac{\tau}{2kh} \qquad \tau_i = 8h$$

Ajuste del parámetro p_{reset}

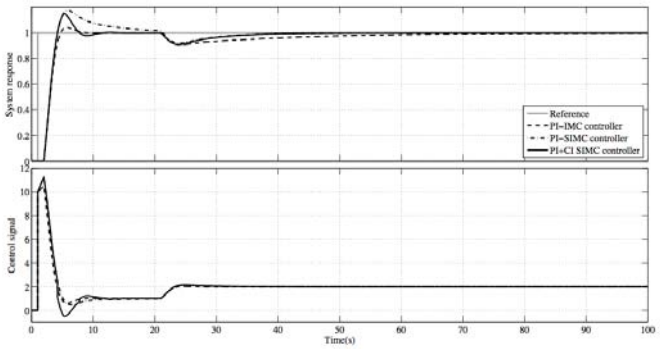
- valores bajos para sistemas “delay-dominant”
- valores medios-altos para sistemas “lag-dominant”
- p_{reset} = 1 para sistemas “integrating”

Sistemas de primer orden con retardo (FOPDT) – “lag dominant”

$$P(s) = \frac{1}{20s + 1} e^{-s}$$

PI base (IMC): kp = 10, ti = 20 s
PI base (SIMC) kp = 10, ti = 8 s
p_{reset} = 0.55

	Reference		Disturbance		Stability margins	
	IAE (s)	ITAE (s^2)	IAE (s)	ITAE (s^2)	φ_m (°)	A_m (dB)
PI-IMC	2.20	5.05	5.85	237.45	61.3	9.92
PI-SIMC	3.09	14.07	2.33	67.2	52.5	9.64
PI+CI-SIMC	2.30	6.47	2.33	67.2	51	9.19



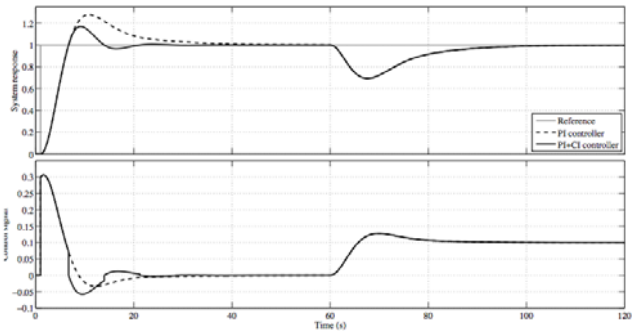
Sistemas de primer orden con retardo (IPDT) – "integrating systems"

$$\tilde{P}(s) = \frac{1}{s} e^{-1.69s}$$

PI base (SIMC): kp = 0.3, ti = 13.5 s

p_{reset} = 1

	Reference		Disturbance		Stability margins	
	IAE (s)	ITAE (s^2)	IAE (s)	ITAE (s^2)	φ_m (°)	A_m (dB)
PI	6.43	62.80	4.48	328.3	48.6	24.28
PI+CI	4.17	20.44	4.48	328.3	48.4	23.9



Sistemas de primer orden con retardo (FOPDT)

System	k_p	τ_i	p_{reset}
First order without delay	$\left(\frac{1+kk_p}{2\tau}\right)^2 < \frac{kk_p}{\tau\tau_i}$ $\alpha = \frac{1+kk_p}{2\tau}, \beta = +\sqrt{\frac{kk_p}{\tau\tau_i} - \alpha^2}$		$p_{reset} = \frac{e^{-\frac{\alpha\pi}{\beta}}}{1+e^{-\frac{\alpha\pi}{\beta}}}$
First order with dominant lag	$\frac{\tau}{2kh}$	$8h$	$[0.3, 0.6]$
First order with dominant delay	$\frac{3\tau}{2kh}$	τ	$--$
Second and higher order	Approximate by a first order plus time delay		
Integrating systems	$\frac{1}{2kh}$	$8h$	1

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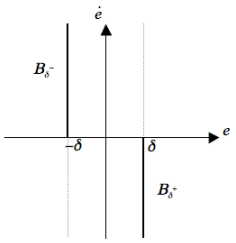
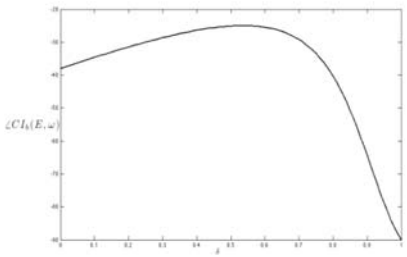
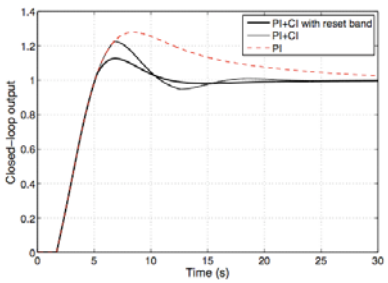
Mejoras del PI+CI

Banda de reset fija

$$C:\begin{cases}\dot{x}_r(t) &= A_r x_r(t)+B_r e(t), & (e(t),\dot{e}(t))\notin B_\delta\\ x_r(t^+) &= A_\rho x_r(t), & (e(t),\dot{e}(t))\in B_\delta\\ v(t) &= C_r x_r(t)+D_r e(t)\end{cases}$$





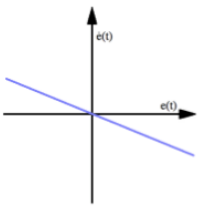


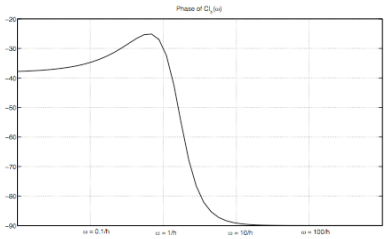
Mejoras del PI+CI

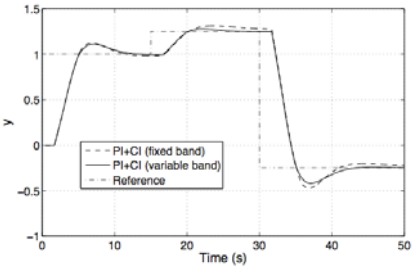
Banda de reset variable

$$C:\begin{cases}\dot{x}_r(t) &= A_r x_r(t) + B_r e(t), & (e(t),\dot{e}(t)) \notin \mathscr{B}_h^v \\ x_r(t^+) &= A_\rho x_r(t), & (e(t),\dot{e}(t)) \in \mathscr{B}_h^v \\ v(t) &= C_r x_r(t) + D_r e(t)\end{cases}$$

$$\mathscr{B}_h^v = \{(e(t),\dot{e}(t)) \in \mathbb{R}^2 \mid h\dot{e}(t) + e(t) = 0\}$$

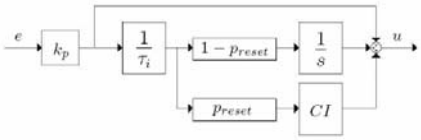


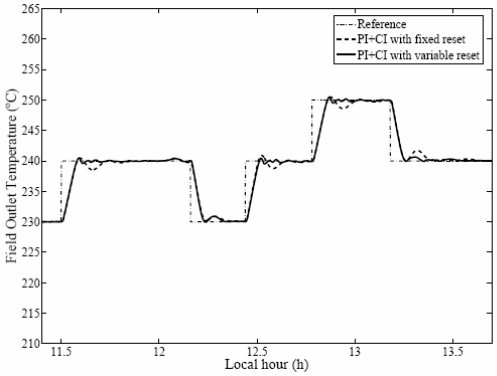




Mejoras del PI+CI

Reset variable: p_{reset} es variable en el tiempo !





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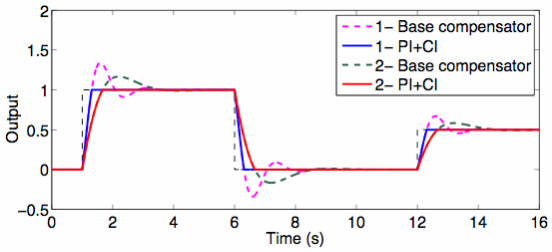
Conclusiones

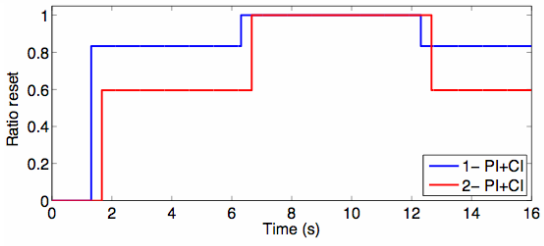
Sistemas de primer orden

$$P: \begin{cases} \dot{x_p}(t) = -a_0 x_p(t) + b_0 v(t) \\ y(t) = x_p(t) \end{cases}$$

$$E_k(s) = \frac{a_0 w_0 - \frac{b_0 k_p x_i(t_k)(1-p_r)}{\tau_i}}{\left(s^2 + (a_0 + b_0 k_p)\, s + \frac{b_0 k_p}{\tau_i}\right)}$$

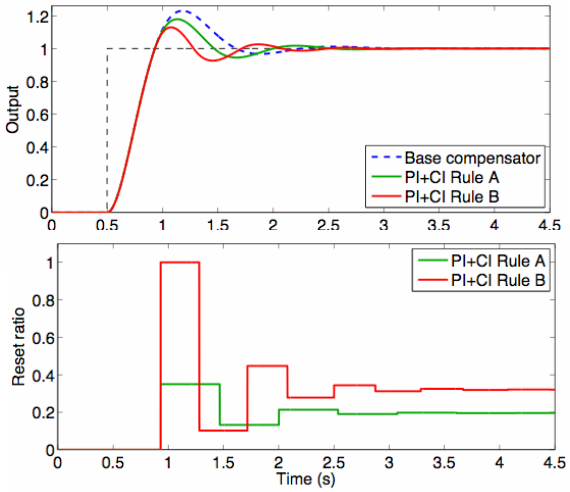
$$p_r(t_k) = 1 - \frac{a_0 \tau_i w_0}{b_0 k_p x_i(t_k^-)}$$



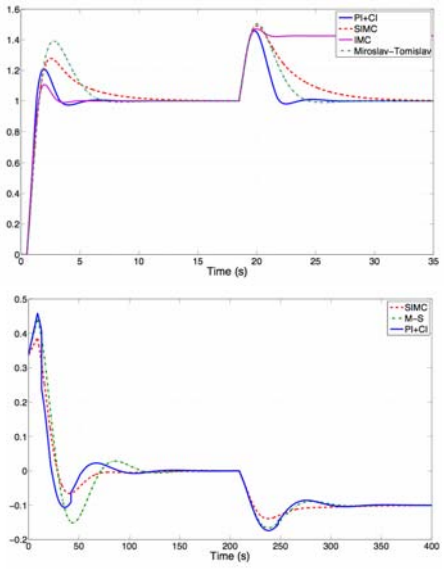


Sistemas de segundo orden

$$P(s) = \frac{b_0}{s^2 + a_1 s + a_0}$$



Sistemas IPDT



- Banda de reset variable
- p_{reset} variable

CONCLUSIONES

- Reset is a simple way to improve control systems performance, overcoming LTI compensation fundamental limitations
- Stability is a main concern, since reset may have a destabilizing effect
- It is not of general use, closed loop response must have overshoot for reset to be effective
- Reset control systems are hybrid systems, modeled as impulsive differential equations
- Reset may be modeled as state-dependent event
- PI+CI has been found effective for lag dominant and integrating systems
- Improvements may be achieved by a variable reset band and a variable reset ratio
- Many open directions for research, both in theory and practice