The Internal Model Principle in tracking periodic signals: concepts and examples of application.

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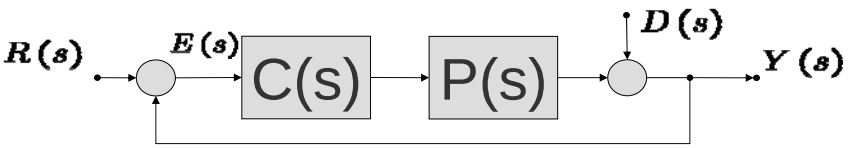
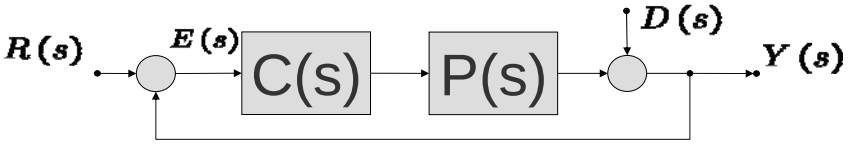
Introduction (I)



The control system goal is to force the output, y(t), to track a certain reference, r(t), without steady state error, e(t).

This should be in the presence of plant uncertainty and disturbances, d(t).

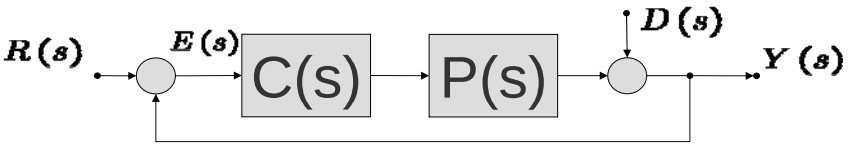
Introduction (II)



Major Design Goals :

- Stability (Robustness)
- Steady State Response (Disturbances)
- Transient Response

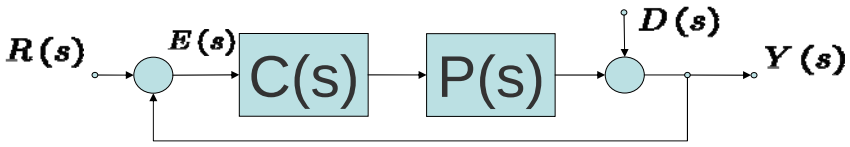
Introduction (III)



$$L(s) = C(s)\,P(s)$$ Open Loop Transfer Function

The open loop transfer function allows to study the closed-loop stability by means of the Nyquist Criteria (Bode Criteria)

Introduction (IV)



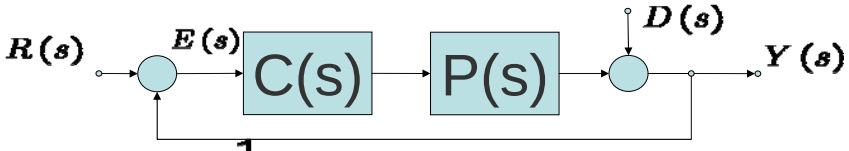
$$T(s) = \frac{L(s)}{1 + L(s)}$$

$$Y(s) = T(s)\,R(s)$$

Complementary Sensitivity Function

In order to be able to track a certain reference it is necessary that T has gain close to 1 in the frequency range contained in R

Introduction (V)



$$S(s) = \frac{1}{1 + L(s)}$$

$$Y(s) = S(s)\,D(s)$$

$$E(s) = S(s)\,R(s)$$

Small $S(s)$

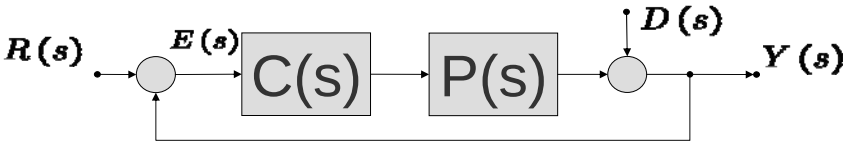
- load disturbance attenuation
- good tracking

Sensitivity Function

In order to be able to reject a certain disturbance, D, it is necessary that S has gain close to 0 in the frequency range contained in D

In order to obtain small error when tracking a signal, R, it is necessary that S has gain close to 0 in the frequency range contained in R

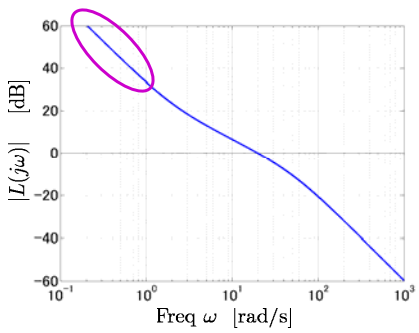
Introduction (VI)

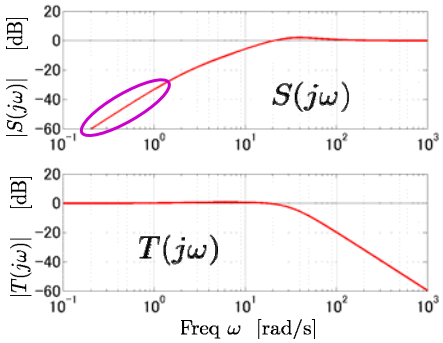


$$T(s) + S(s) = 1$$

This relationship introduces a constraints in the tracking/rejecting capabilities

Introduction : (Example I)





At low frequencies

$$|L(j\omega)| > 101 \longrightarrow |S(j\omega)| = \left|\frac{1}{1+L(j\omega)}\right| < \frac{1}{100}$$

40dB −40dB

- Load disturbances will be attenuated by a factor of 100
- Tracking error is less than 1%

Introduction (VIII)

$$S(s) + T(s) = \frac{1}{1+L} + \frac{L}{1+L} = 1$$

- We cannot make both $S(s)$ and $T(s)$ small simultaneously.
- Divide frequency range in which they are small.

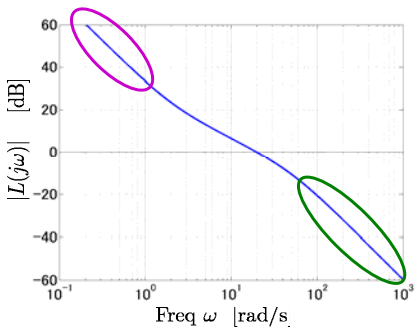
•Load disturbance
•Tracking
} at low frequency

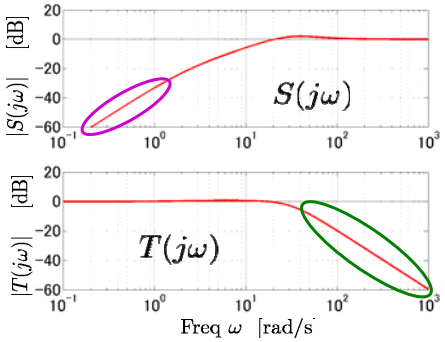
•Noise } at high frequency

Introduction (IX)

- In order to obtain the specifications
  - T(s) must be small at the low frequency range
  - S(s) must be small at the high frequency range
- It is necessary that
  - L(s) is big at the low frequency range
  - L(s) is small at the high frequency range

Introduction (Example II)





Introduction (X)

$$S(s) = \frac{1}{1 + L(s)} = \frac{D_c(s)\, D_p(s)}{D_c(s)\, D_p(s) + N_c(s)\, N_p(s)}$$

$$Y(s) = S(s)\, \frac{N_D(s)}{D_D(s)}$$

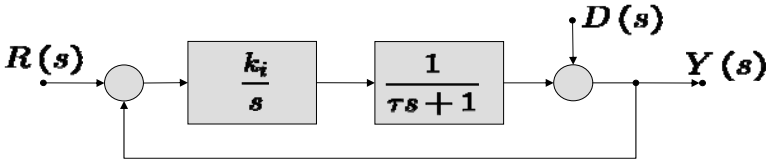
Rejecting disturbances implies low gain (i.e. open loop poles) at the desired frequency range.

$$T(s) = \frac{L(s)}{1 + L(s)} = \frac{N_c(s)\, N_p(s)}{N_c(s)\, N_p(s) + D_c(s)\, D_p(s)}$$

$$Y(s) = T(s)\, \frac{N_R(s)}{D_R(s)}$$

Tracking references implies high gain (i.e. open loop zeros) at the desired frequency range.

Introduction : Example III



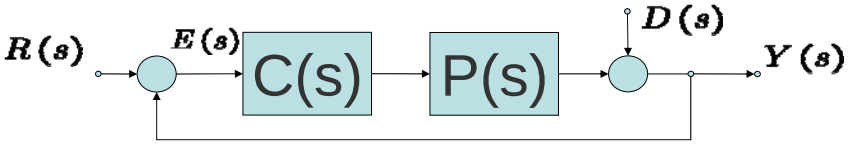
$$R(s)=\frac{k}{s} \quad D(s)=\frac{a}{s}$$

$$S(s)=\frac{s(\tau s+1)}{k_i+s(\tau s+1)} \longrightarrow \lim_{s\to 0} s\cdot\frac{s(\tau s+1)}{k_i+s(\tau s+1)}\frac{a}{s}=0$$

$$T(s)=\frac{k_i}{k_i+s(\tau s+1)} \longrightarrow \lim_{s\to 0} s\cdot\frac{k_i}{k_i+s(\tau s+1)}\frac{k}{s}=T(0)\,k=k$$

Type concept (I)

- In classical control introduction courses it is analyzed how a certain system must be in order to track certain type of signals.



$$E(s)=\frac{1}{1+C(s)P(s)}R(s)$$

$$\lim_{t\to\infty} e(t)=\lim_{s\to 0} s\cdot E(s)=\lim_{s\to 0} s\cdot\frac{1}{1+C(s)P(s)}R(s)$$

The steady state error depends on reference signal.

Type concept (II)

IMP in control

$$R(s) = \frac{1}{s} \qquad \lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot E(s) = \lim_{s\to 0} \cdot \frac{1}{1 + C(s)P(s)} = \cdot \frac{1}{1 + \lim_{s\to 0} C(s)P(s)}$$

In order to be 0 it is necessary that C(s)P(s) contains a pole in s=0

$$R(s) = \frac{1}{s^2} \qquad \lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot E(s) = \lim_{s\to 0} \cdot \frac{1}{(1 + C(s)P(s)) \cdot s} = \cdot \frac{1}{\lim_{s\to 0} s \cdot C(s)P(s)}$$

In order to be 0 it is necessary that C(s)P(s) contains two poles in s=0

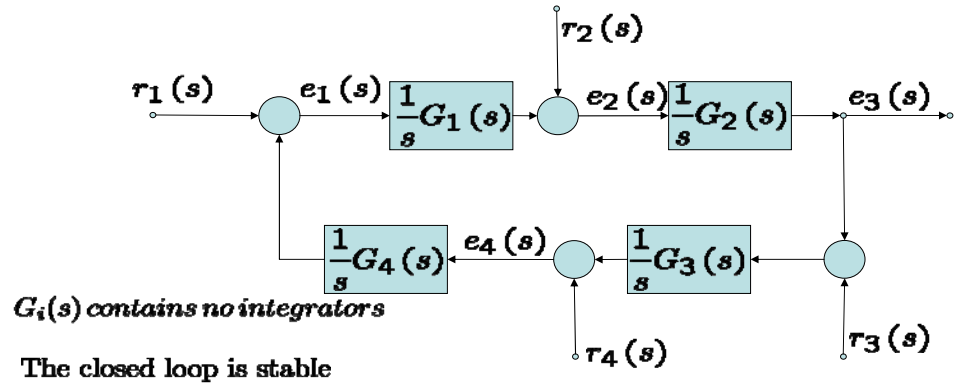
Type : Number of integrators in the open-loop transfer functions

Type concept (III)

IMP in control

Input	Steady-state error formula	Type 0		Type 1		Type 2	
		Static error constant	Error	Static error constant	Error	Static error constant	Error
Step, $u(t)$	$\frac{1}{1+K_p}$	K_p = Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	K_v = Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	K_a = Constant	$\frac{1}{K_a}$

Type concept (IV)

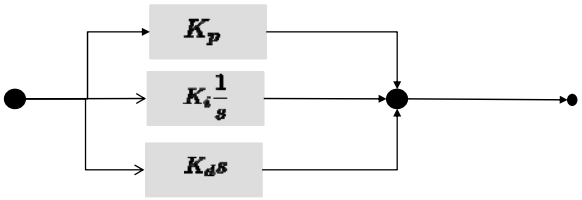


$G_i(s)$ *contains no integrators*

The closed loop is stable

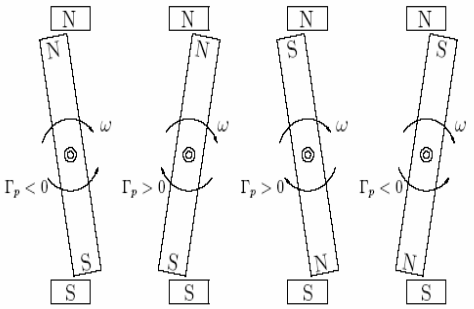
- Type I : $e_1 - r_2, e_2 - r_3, e_3 - r_4, e_4 - r_1$
- Type II : $e_1 - r_3, e_2 - r_4, e_3 - r_1, e_4 - r_2$
- Type III : $e_1 - r_4, e_2 - r_1, e_3 - r_2, e_4 - r_3$
- Type IV : $e_1 - r_1, e_2 - r_2, e_3 - r_3, e_4 - r_4$

PID control limitation (I)



- The PID controller is one of the most popular controllers.
  - Contains a integrator (Type I). It allows to track steps in steady state
  - Three independent design parameters (Kp,Ki,K_d)
  - Easy to tune for simple plants.

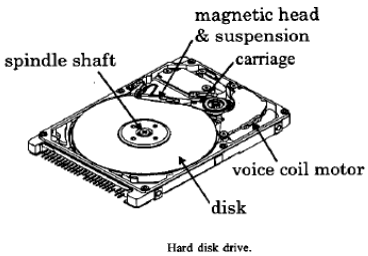
PID control limitation (II)



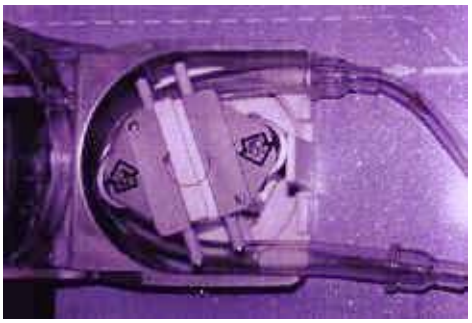
Virtual Laboratory

PID Control limitations (III)

- In practice, many real systems have to handle tracking and rejecting periodic signals.



Hard disk drive.



Peristaltic Pump

PID Control limitations (IV)

- In power Electronics
  - Rectifier : Generating dc signal from a 50/60 Hz signal
  - Inverter : Generating a 50/60 Hz signal from dc one (Tracking a reference signal)
  - Active filter : Compensation of harmonic signals (Rejecting periodic signals)

Internal model principle (I)

- **A key** topic in classical control theory is the Internal Model Principle (IMP).
- This principle states that if a certain signal must be tracked or rejected without steady-state error, the generator must be inside the control loop, in the controller, or in the plant itself.

Internal model principle (II)

$$E(s) = S(s)\,R(s) = \frac{1}{1+L(s)}R(s) = \frac{D_c(s)\,D_p(s)}{D_c(s)\,D_p(s)+N_c(s)\,N_p(s)}\frac{N_R(s)}{D_R(s)}$$

- The error vanishes asymptotically if:
  - r(t) ! 0 as t ! 1 . (This case it is not important from the steady state time response viewpoint, only affects transient response)
  - $D_R(s)$ is contained in $Dc(s)D_p(s)$. IMP application.

Internal model principle (III)

- Standard classical control subjects include the IMP concept implicitly when they introduce the system-type concept.
- The type concept can only be applied to polynomial signals (step, ramp, and parabola) whose generator has the form in the Laplace domain.

$$\mathcal{L}(g(t)) = \frac{1}{s^n}$$

Internal model principle (IV)

$e(t)$	$E(s)$	$E(z)$
$u_s(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$
$t \cdot u_s(t)$	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$
$\frac{t^2}{2} \cdot u_s(t)$	$\frac{1}{s^3}$	$\frac{T^2 z(z+1)}{2(z-1)^3}$
e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$	$\frac{z\sin(aT)}{z^2-2z\cos(aT)+1}$
$\cos(at)$	$\frac{s}{s^2+a^2}$	$\frac{z(z-\cos(aT))}{z^2-2z\cos(aT)+1}$

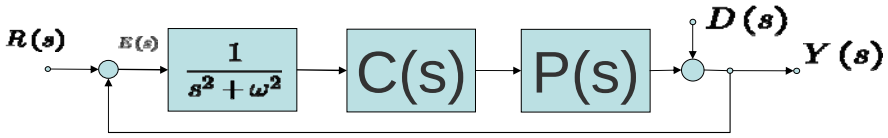
Some References

- Internal Model Principle in control theory. B. Francis and W. Wonham, *Automatica*, vol. 12, pp. 457–465, 1976.
- Feedback Control Theory, J. C. Doyle, B. A. Francis, and A. R. Tannenbaum, Macmillan, New York, 1992.

Introduction (I)

- Periodical Signals are present in many real world applications.
- Periodical Signals can be described as a linear combination of sinusoidal signals (sin,cos).
- According to the IMP necessary and sufficient condition to asymptotically track or reject signals in a closed-loop system is that open-loop transfer function contains the signal generator.
- We are interested in tracking and rejecting sinusoidal signals. The linear generator of this type of signals, commonly named resonator, is characterized by having a denominator $s^2+!^2$.

Introduction (II)

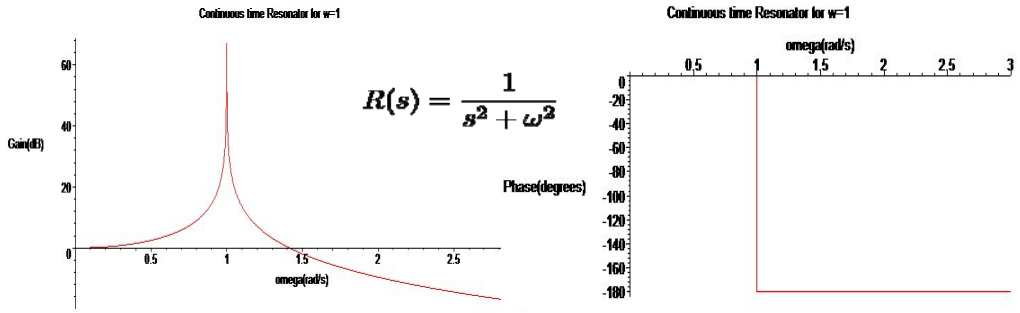


$$\frac{Y(s)}{R(s)} = \frac{N_C(s)N_P(s)}{N_C(s)N_P(s) + D_C(s)D_P(s)\left(s^2+\omega^2\right)}$$

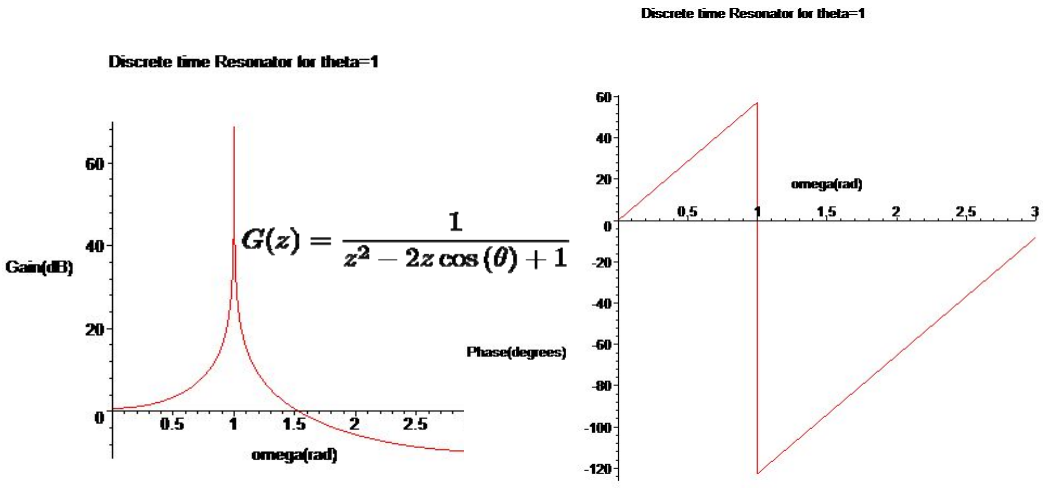
$$\frac{E(s)}{R(s)} = \frac{D_C(s)D_P(s)\left(s^2+\omega^2\right)}{N_C(s)N_P(s) + D_C(s)D_P(s)\left(s^2+\omega^2\right)}$$

Introduction (III)

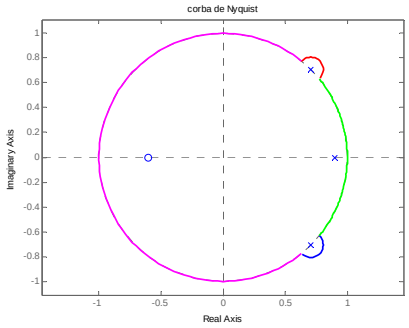
- The IMP only fixes the internal model denominator.
- Using simply a denominator introduces an important phase lag so the system will be difficult to stabilize.

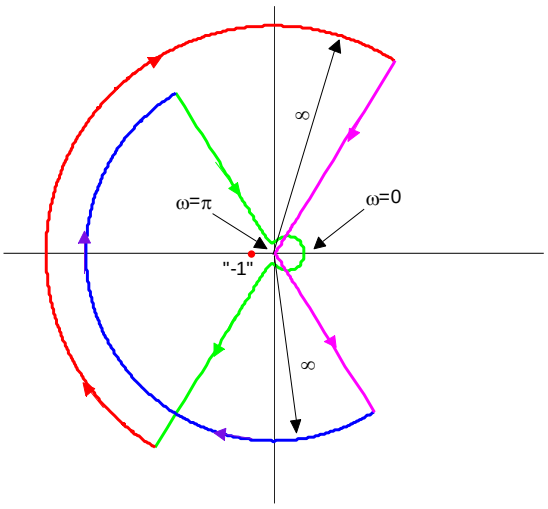


Introduction (IV)



Exemple



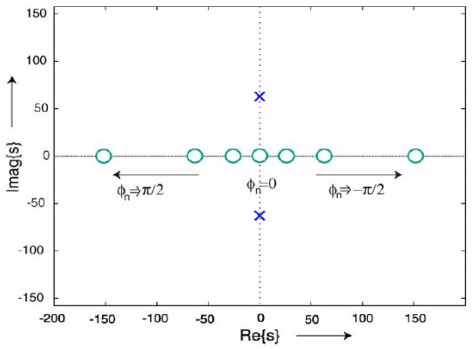


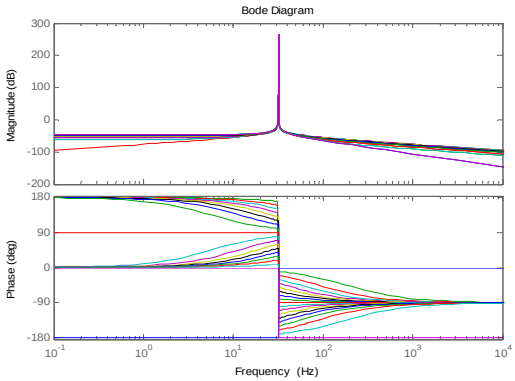
AFC Controllers (I)

- AFC : Adaptive Feedforward Cancellation
- It is possible to fix the resonators zeroes so that the resonator fixes a certain phase at the interest frequency.
- This procedure allows to guarantee closed-loop stability.

AFC Controllers (II)

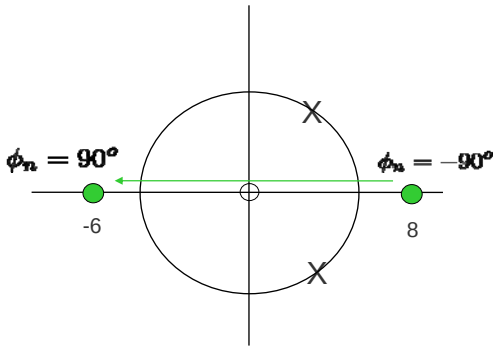
$$R_n(s) = g_n \frac{s\cos(\phi_n) + \omega_n \sin(\phi_n)}{s^2 + \omega_n^2} \qquad s = -\omega_n \tan(\phi_n)$$

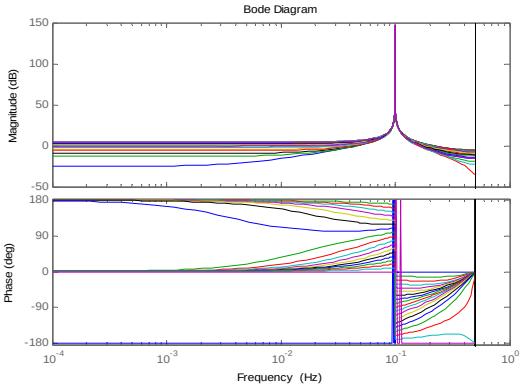




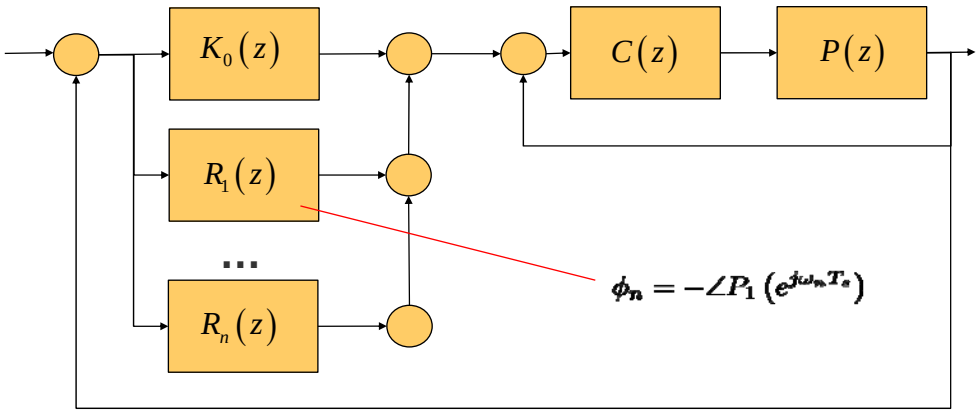
AFC Controllers (IV)

$$R_n(z) = g_n \frac{\cos(\phi_n)\, z^2 - \cos(\omega_n T_s + \phi_n)\, z}{z^2 - 2\cos(\omega_n T_s)\, z + 1} \qquad z = 0,\ z = \frac{\cos(\omega_n T_s + \phi_n)}{\cos(\phi_n)}$$

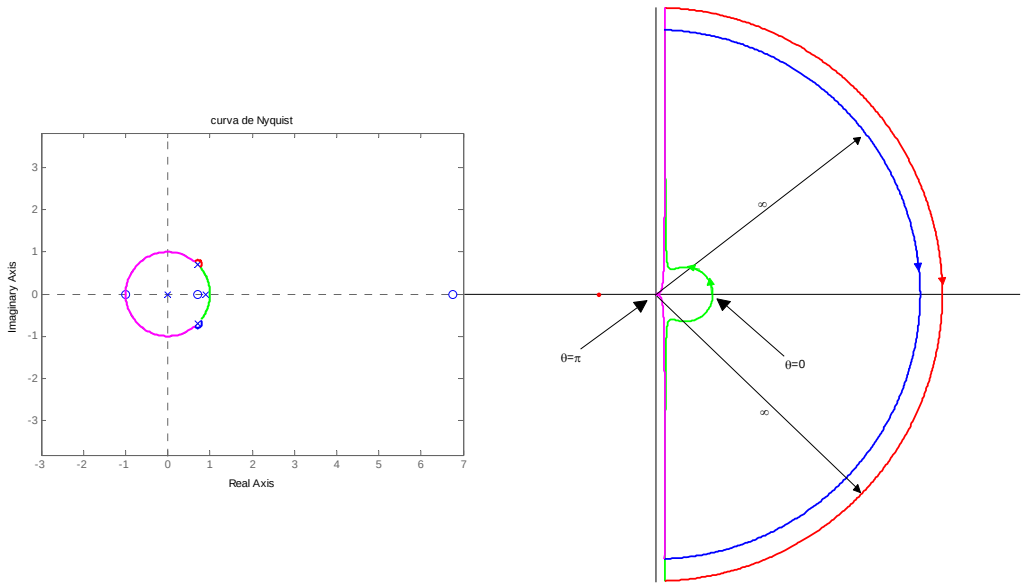




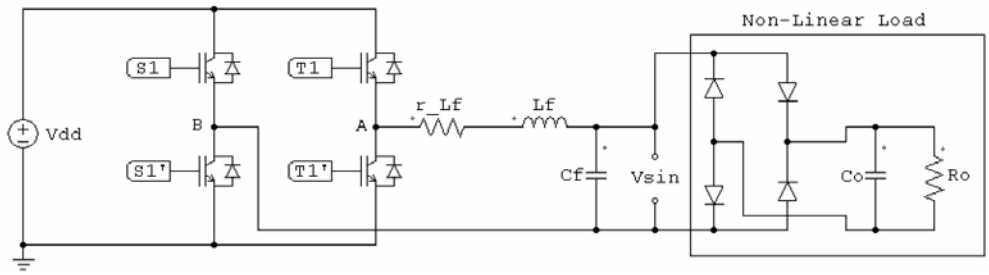
AFC Controllers (VI)



Example



Problem formulation



Control Goal

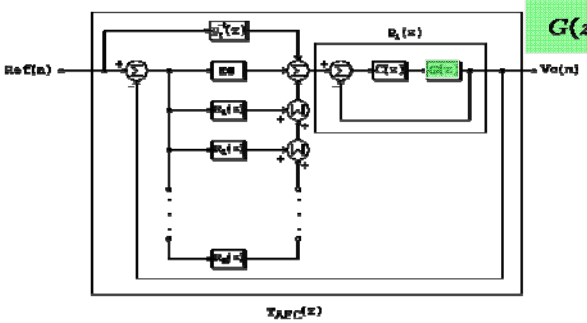
$$V_{\sin}(t) = E \cdot \sin(\omega t)$$

Controller Design I

$$G(z) = \frac{V_o(z)}{U(z)} = 3\{ZOH(s)G(s)\}$$

$$\Downarrow$$

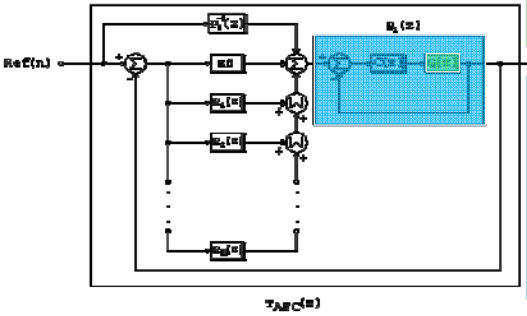
$$G(z) = \frac{16.4625(z + 0.97)}{z(z^2 - 1.837z + 0.9129)}$$



$V_{dd} = 425$ V $\quad L_f = 384.00\ \mu$H
$r_{L_f} = 700$ mΩ $\quad C_f = 81.00\ \mu$F
$C_o = 6400\ \mu$F $\quad R_o = 24\Omega$

$T = 1/f_s = 1/20000 = 50\mu$s

Controller Design II



$$G(z) = \frac{V_o(z)}{U(z)} = 3\{ZOH(s)G(s)\}$$

$$\Downarrow$$

$$G(z) = \frac{16.4625(z+0.97)}{z(z^2-1.837z+0.9129)}$$

$$C(z) = \frac{0.0098(z^2-1.837z+0.9129)}{(z-0.856)(z-0.078)}$$

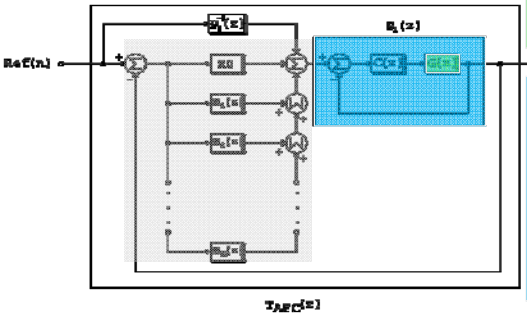
$G.M. = 9.3$ dB and $P.M. = 47°$.

$$P_1(z) = \frac{0.16133(z+0.97)}{(z+0.2774)(z^2-1.211z+0.5641)}$$

$V_{dd} = 425$ V $\quad L_f = 384.00\ \mu$H
$r_{L_f} = 700\ \text{m}\Omega \quad C_f = 81.00\ \mu$F
$C_o = 6400\ \mu$F $\quad R_o = 24\Omega$

$T = 1/f_s = 1/20000 = 50\mu$s

Controller Design III



$$G(z) = \frac{V_o(z)}{U(z)} = 3\{ZOH(s)G(s)\}$$

$$\Downarrow$$

$$G(z) = \frac{16.4625(z+0.97)}{z(z^2-1.837z+0.9129)}$$

$$C(z) = \frac{0.0098(z^2-1.837z+0.9129)}{(z-0.856)(z-0.078)}$$

$G.M. = 9.3$ dB and $P.M. = 47°$.

$$P_1(z) = \frac{0.16133(z+0.97)}{(z+0.2774)(z^2-1.211z+0.5641)}$$

$V_{dd} = 425$ V $\quad L_f = 384.00\ \mu$H
$r_{L_f} = 700\ \text{m}\Omega \quad C_f = 81.00\ \mu$F
$C_o = 6400\ \mu$F $\quad R_o = 24\Omega$

$T = 1/f_s = 1/20000 = 50\mu$s

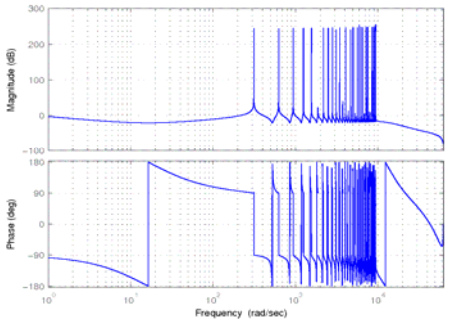
$$C_{AFC}(z) = K_0 + \sum_{k=1}^{30} R_k(z)$$

$K_0 = 0.01 \qquad \omega_k = 2\pi 50k$

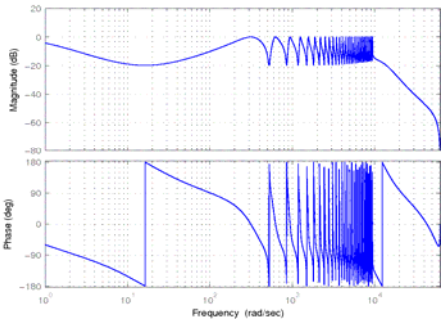
$G.M. = 17.80$ dB and $P.M. = 82.58°$.

$P_1^{-1}(z) = 1/(P_1(z_1))$ with $z_1 = e^{j\omega_1 T}|_{\omega_1 = 2\pi 50}$

Controller Design IV



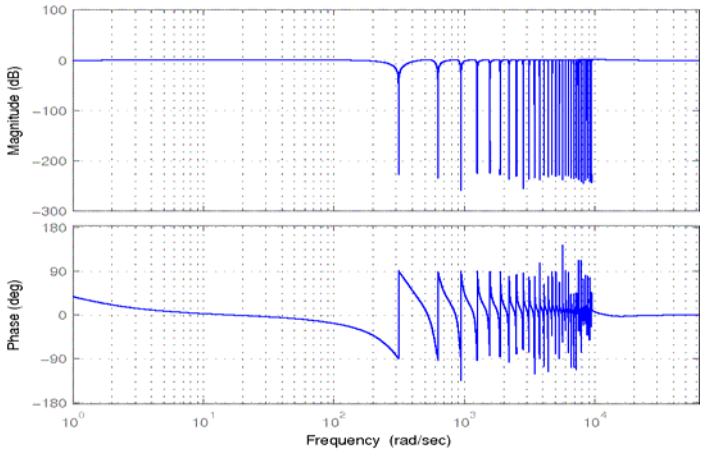
AFC open-loop Bode plot for the DC-AC inverter system.



AFC Closed-loop Bode plot for the DC-AC inverter system.

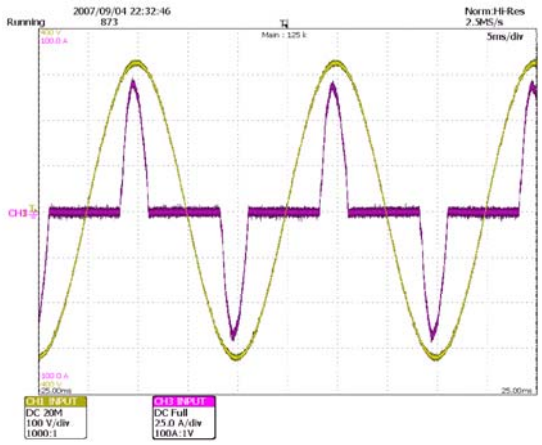
30 harmonics

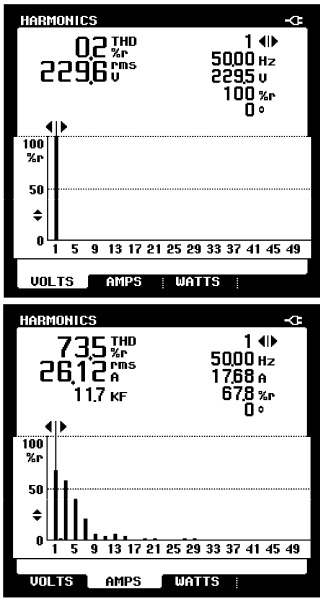
Controller Design V



Closed-loop Bode plot for the AFC sensitivity transfer function.

Experimental results





Output voltage and current shapes for the inverter feeding the nonlinear load at 4.0 kVA.

Final comments

Virtual Laboratory

Some References

- Design of Adaptive Feedforward Controllers Using Internal Model Equivalence. William Messner & Marc Bodson. Proceedings of the American Control Conference. Baltimore, Maryland (June 1994)
- A General Theory of Linear Time-Invariant Adaptive Feedforward Systems with Harmonic Regressors. David S. Bayard. IEEE Transactions on Automatic Control, Vol. 45, NO. 11, November 2000
- A loop shaping perspective for tuning controllers with adaptive feedforward cancellation. F. Byl Marten, Stephen J. Ludwick, David L. Trumper. Precision engineering 2005, vol. 29, no1.
- Adaptive Feed-Forward Cancellation Control of a Full-Bridge DC-AC Voltage Inverter. Shane Malo and Robert Griñó. Proceedings of the 17th IFAC World Congress . Seoul, Korea, July 6-11, 2008
- Resonant Control of a Single-Phase Full-Bridge Unity Power Factor Boost Rectifier. Ramon Costa-Castelló, Robert Griñó, Enric Fossas. 2007 IEEE Multi-conference on Systems and Control .Suntec City Convention Centre. Singapore. October 1-3, 2007.

Internal Model (I)

- Any Periodical system can be written by means of its Fourier Series:

$$r(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\cdot k\cdot \pi\cdot t}{Tp}}$$

- Its generator can be written as :

$$R(s) = \frac{1}{s}\prod_{n=1}^{\infty}\frac{\left(\frac{2n\pi}{Tp}\right)^2}{s^2+\left(\frac{2n\pi}{Tp}\right)^2}$$

Internal Model (II)

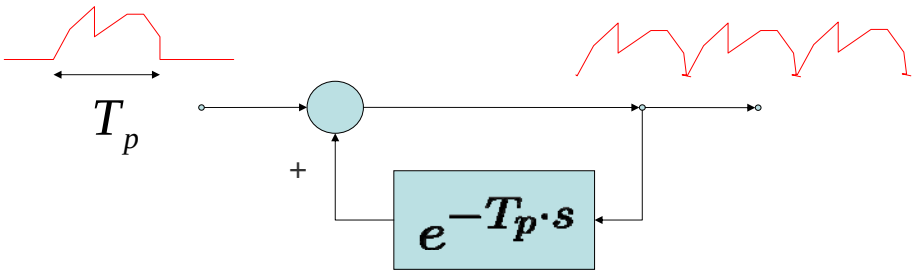
$$R(s) = \frac{1}{s} \prod_{n=1}^{\infty} \frac{\left(\frac{2n\pi}{T_p}\right)^2}{s^2 + \left(\frac{2n\pi}{T_p}\right)^2}$$

- Infinite terms
- Difficult to implement

$$R(s) = \frac{T_p \cdot e^{\frac{-T_p \cdot s}{2}}}{1 - e^{-T_p \cdot s}}$$

- Compact Expression

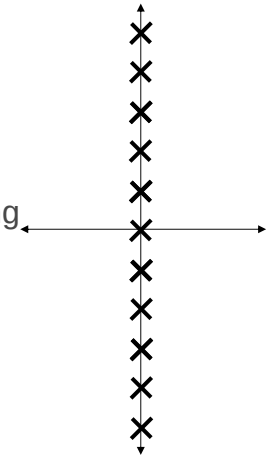
Internal Model (III)



- This temporal response introduces the term “repetitive control”
- This behavior relates repetitive control with “learning Control”
- Similar to Iterative Learning Control (ILC)

Internal Model (IV)

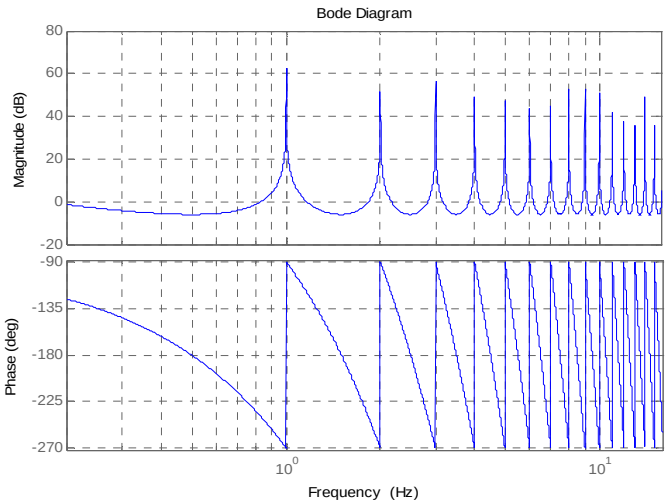
$$R(s) = \frac{1}{1 - e^{-T_p \cdot s}}$$



- Contains infinite poles over the imaginary axis
- These pole introduce infinite gain at the following frequencies:

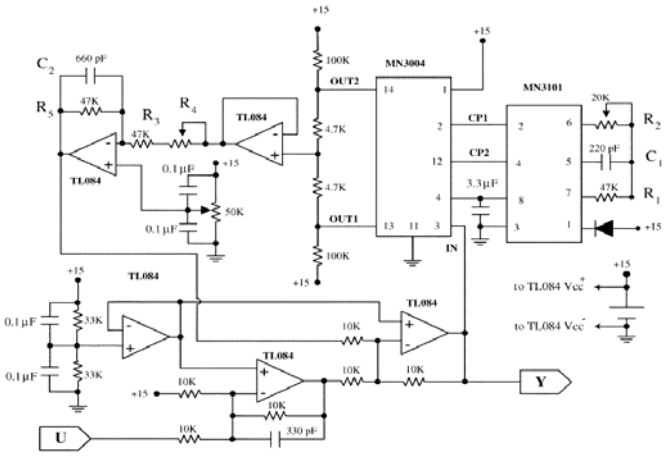
$$\omega_k = \frac{2\pi \cdot k}{T_p}$$

Internal Model (V)



$T_p = 1s$

Internal Model (VI)



IC MN3004 : a 512-stage low-noise bucket brigade delay BBD, in the range [2.5,25] ms.

Reproduced from : Analog Circuits to Implement Repetitive Controllers for Tracking and Disturbance Rejection of Periodic Signals. J. Leyva-Ramos, *G. Escobar, P. R. Martínez, and P. Mattavelli.* IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—II: EXPRESS BRIEFS, VOL. 52, NO. 8, AUGUST 2005

Internal Model (VII)

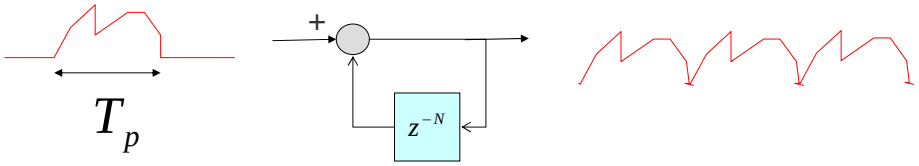
$$R(s) = \frac{1}{1 - e^{-T_p \cdot s}}$$

- This transfer function contains a pure time delay so it is difficult to implement in continuous time

$$R(z) = \frac{1}{1 - z^{-N}}$$

- A time delay in discrete time is very simple to implement in discrete time

Internal Model (VIII)



$$T_p = NT_s$$

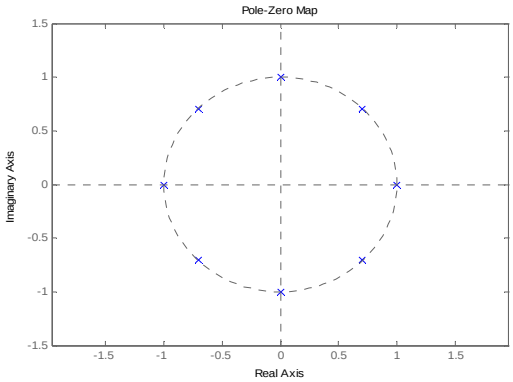
- The time response is similar to the continuous time one
- Poles are placed over the unity circle (equidistributed)
- Infinite gain at frequencies :

$$R(z) = \frac{1}{1 - z^{-N}}$$

$$z = \sqrt[N]{1}$$

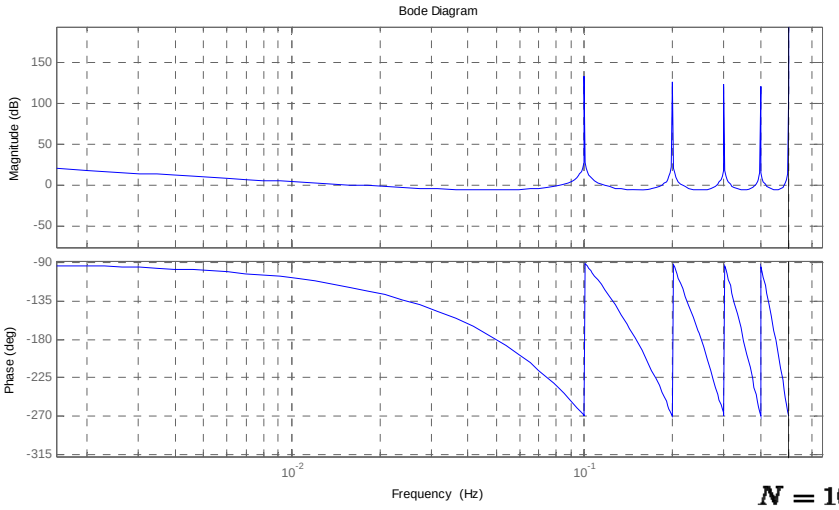
$$\omega_k = \frac{2\pi k}{NT_s}$$

Internal Model (IX)



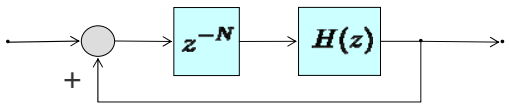
$$\frac{1}{z^8 - 1}$$

Internal Model (X)



$N = 10$

Internal Model (XI)



- In practical applications it is necessary to reduce the gain at high frequencies (uncertainty in the model)
- In order to reduce the gain a low pass filter is introduced in the internal model

Internal Model (XII)

- Usually a low pass null-phase FIR filter is used.

$$H(z) = q_1 \cdot z + q_0 + q_1 \cdot z^{-1}$$

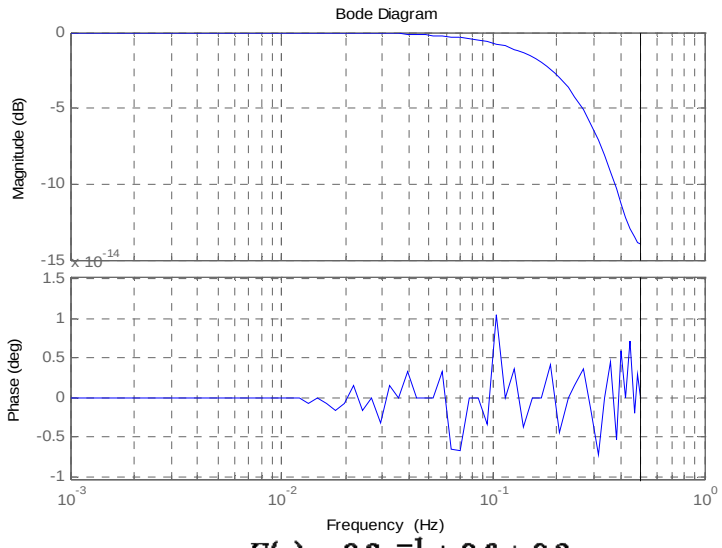
  - Order filters (ie. Butterworth, degradates the system performance)

- In order to preserve the integrator it is necessary that gain at continuous frequency is 1

$$H(1) = q_0 + 2 \cdot q_1 = 1$$

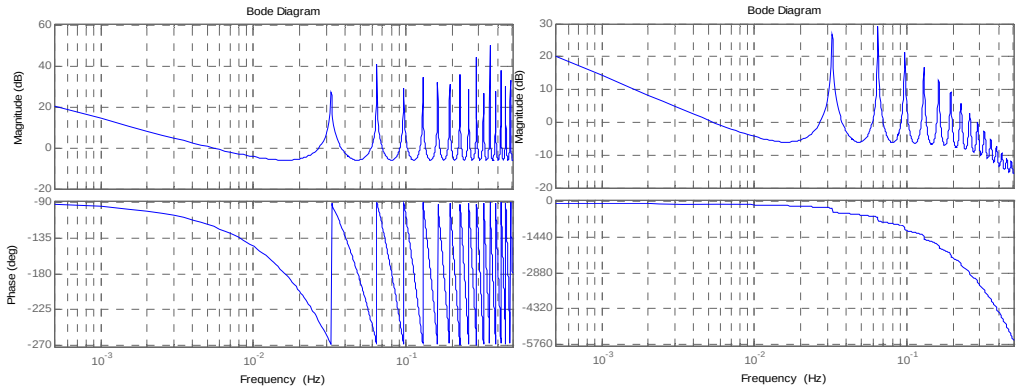
- No causality problems exist due to the fact that the filter is in series connection with a delay (N periods).
- The filter reduces the internal model gain at higher frequencies (robustness). Unfortunately the filter slightly changes the poles frequencies (performance loose).

Internal Model (XII)



$$F(z) = 0.2z^{-1} + 0.6 + 0.2z$$

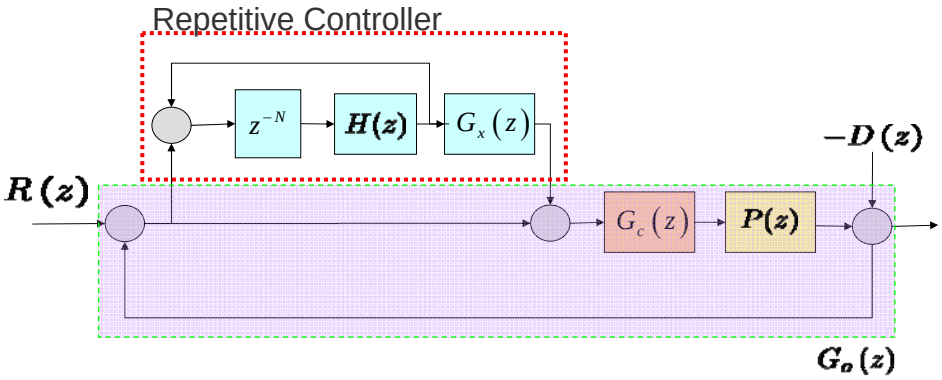
Internal Model (XIV)



Stabilizing techniques (I)

- Repetitive control Internal Models are :
  - High order .
  - Complex frequency response.
  - As a consequence it is difficult to study the closed-loop stability
    - Closed-loop poles
    - Nyquist Criteria
- Most schemes need a stabilizing controller
- Predefined Architectures are defined
  - Series connection
  - P*lug-in Connection*
  - Passivity based methods

Stabilizing techniques (III)



Stabilizing techniques (IV)

Complete System Sensitivity function

$$S(z) = \frac{E(z)}{R(z)} = S_o(z) S_{Mod}(z)$$

Sensitivity function without repetitive control

$$S_o(z) = \frac{1}{1 + G_c(z)\, G_p(z)}$$

Modifying Sensitivity function

$$S_{Mod}(z) = \frac{1 - \sigma W(z) H(z)}{1 - \sigma W(z) H(z) (1 - G_x(z) G_o(z))}$$

Stabilizing techniques (V)

- Stability conditions:

1. The closed-loop system without the repetitive controller must be stable
2. Low-pass filter must have a gain less than one $\longrightarrow G_c(z)$

$$\|H(z)\|_\infty < 1$$

3. The following condition must be fullfilled

$$\|1 - \frac{G_o(z)}{1+G_o(z)} G_x(z)\|_\infty < 1 \longrightarrow G_x(z)$$

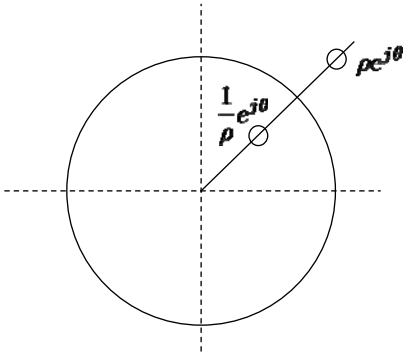
Stabilizing techniques (VII)

$$N^-(z) = (z-p)$$

$$C(z) = N^-\left(\frac{1}{z}\right) = \frac{1-pz}{z}$$

$$\longrightarrow C(e^{j\theta})N^-(e^{j\theta}) = -2p\cos(\theta) + 1 + p^2$$

Phase cancellation scheme



Stabilizing techniques (VII)

- The most direct approach to design $G_x(z)$ is:

$$G_x(z) = k_r \left(\frac{G_o(z)}{1 + G_o(z)} \right)^{-1}$$

- Nonminimum phase plants:

$$G_x(z) = k_r \frac{D(z) N^-\left(z^{-1}\right)}{N^+(z) b_{max}}$$

- k_r is selected using a trade-off between time response and robustness.

Example

Virtual Laboratory

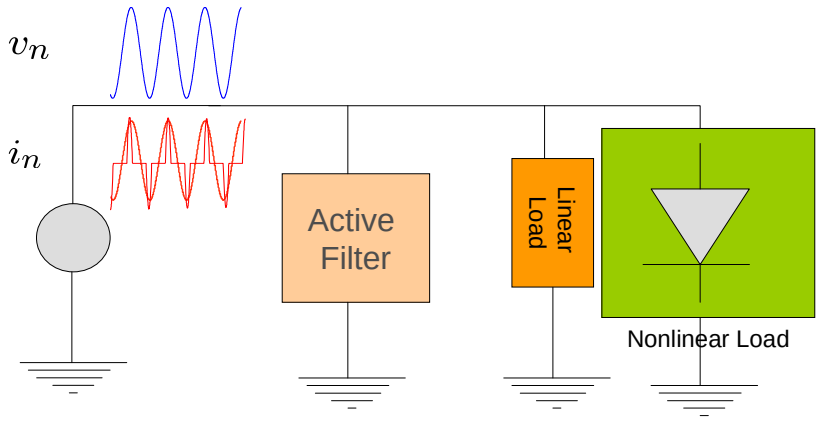
Some References

- ***Digital repetitive plug-in controller for odd-harmonic periodic references and disturbances*** Robert Griñó and Ramon Costa-Castelló. Automatica. Volume 41, Issue 1,Pages 153-157(January 2005)
- ***A New Passive Repetitive Controller For Discrete-Time Finite-Frequency Positive-Real Systems.*** R. Costa-Castelló, D. Wang, R. Griñó. CDC'06
- ***A Repetitive Controller for Discrete-Time Passive Systems.*** Ramon Costa-Castelló, Robert Griñó. Automatica. Volume 42, Issue 9,Pages 1605-1610 (September 2006)
- ***A Repetitive-Based Controller for the Compensation of 6l†1 Harmonic Components***. G. Escobar, *P.G. Hernandez-Briones, P. R. Martinez,* M. Hernandez-Gomez, and R. E. Torres-Olguin. IEEE Transactions on Industrial Electronics, Vol. 55, No. 8, August 2008
- **Fractional Delay Based Repetitive Control with Application to PWM DC/AC Converters** *Yigang Wang; Danwei Wang; Bin Zhang; Keliang Zhou. Control Applications 2007. CCA 2007. IEEE International Conference on 1-3 Oct 2007. Pages(s) 928-933*
- **Repetitive control for systems with uncertain period-time**. Maarten Steinbuch. Automatica 38 (2002) 2103 – 2109
- **Design of noise and period-time robust high-order repetitive control, with application to optical storage**. Maarten Steinbucha*,, SiepWeilandb, Tarunraj Singhc.* Automatica 43 (2007) 2086 – 2095
- **Adaptive Repetitive Control to Track Variable Periodic Signals With Fixed Sampling Rate**. Zhenwei Cao and Gerard F. Ledwich*, Senior Member, IEEE.* IEEE/ASME TRANSACTIONS ON MECHATRONICS, VOL. 7, NO. 3, SEPTEMBER 2002

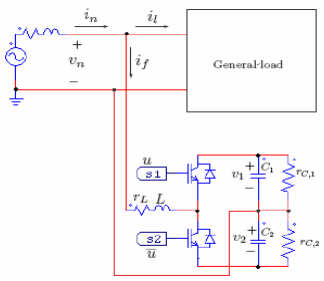
Problem presentation I

- Nonlinear loads in AC electrical networks imply periodic nonsinusoidal currents.
- These currents cause low energy quality (distorted voltages) and poor network utilization.
- To minimize this problem: passive filters and/or active filters.
- Active filters to compensate distorted currents: shunt connected.

Problem presentation II



Problem presentation III



$$\begin{cases} L\dfrac{di_f}{dt} &= -r_L i_f - v_1 d - v_2(d-1) + v_n \\ C_1\dfrac{dv_1}{dt} &= -\dfrac{v_1}{r_{C,1}} + i_f d \\ C_2\dfrac{dv_2}{dt} &= -\dfrac{v_2}{r_{C,2}} + i_f(d-1) \end{cases}$$

$$v_n = V_n\sqrt{2}\sin(\omega_n t); \quad d \in [0,1]$$

$$i_n = i_f + i_l$$

Load description: $i_l = \sum_{n=0}^{\infty} a_n \sin(\omega_n (2\cdot n+1)\, t) + b_n \cos(\omega_n (2\cdot n+1)\, t)$

$a_n, b_n \in \mathbb{R}$ Only odd harmonics.

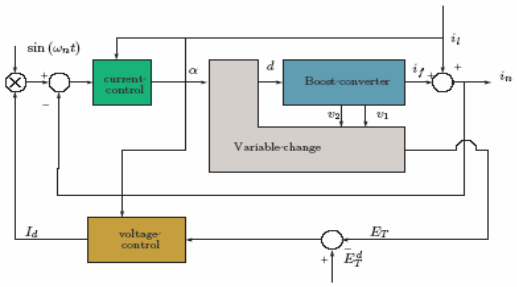
Active filter topology: single-leg split DC bus boost converter.

Controller design I

IMP in control

- Sinusoidal current (in-phase) in the network port (PF=1): $i_n{}^* = I_d^* \sin(\omega_n t)$
- Constant average voltage value in the DC bus:

$$< v_1 + v_2 >_0^* = 2v_d; \quad v_d > \sqrt{2}V_n$$



- Current loop: disturbance attenuation problem.
- Sensors: network current (variable to be controlled), load current (feedforward path), network voltage and DC bus voltages.
- To control the DC bus voltage is equivalent to balance the active power of the filter + the load.

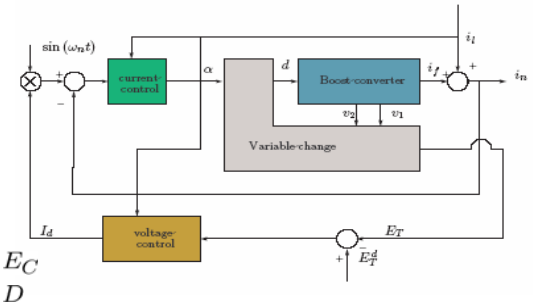
Controller design II

IMP in control

- Partial state feedback: $\alpha = v_1 \cdot d + v_2 \cdot (d-1)$
- Change of variables: $i_f = i_f,\ E_C = \frac{1}{2}\left(C_1 v_1^2 + C_2 v_2^2\right),$ $D = C_1 v_1 - C_2 v_2$

Assuming $C = C_1 = C_2,\ r_C = r_{C,1} = r_{C,2}$ results:

$$\begin{cases} L\dfrac{di_f}{dt} &= -r_L i_f + v_n - \alpha \\ \dfrac{dE_c}{dt} &= -\dfrac{2E_c}{r_C \cdot C} + i_f \cdot \alpha \\ \dfrac{dD}{dt} &= -\dfrac{1}{r_C \cdot C} D + i_f \end{cases}$$



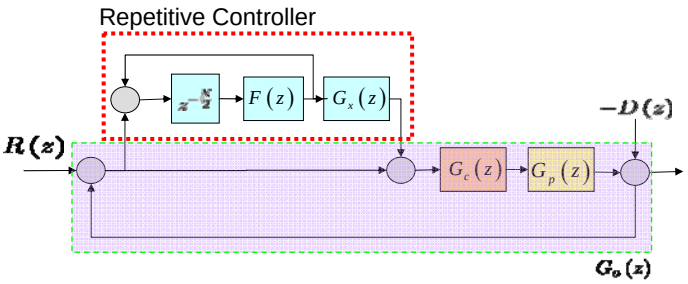
Energy stored in the capacitors: E_C

Charge unbalance: D

Controller design III

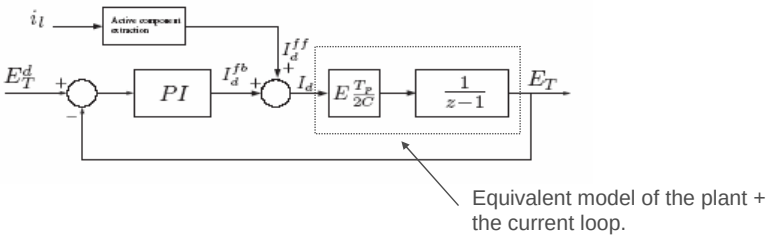
- Digital odd-harmonic repetitive control (plug-in) + feedforward path.

  ➡ rejection of periodic disturbances and tracking of periodic references.



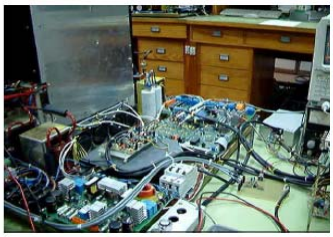
Controller design IV

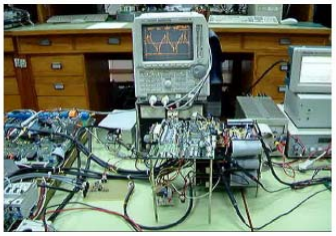
- Digital PI controller + feedforward path (active part of the fundamental component of the load current).



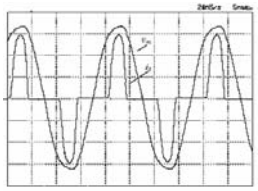
- The PI feedback controller compensates the dissipation and the uncertainties.
- The feedforward path supplies information from the load about the necessary network current.

Experimental results I



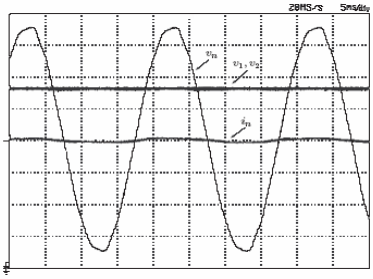


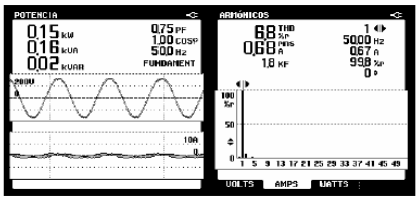
Rectifier current



- **Active filter:** half-bridge boost converter (split DC bus), IGBT switches (100 A), switching frequency 20 kHz, regular centered PWM.
- **Nonlinear load:** full-wave diode rectifier, DC power 4.5 kW, current THD=64%, Irms=26.6 A.
- **Control hardware:** floating point DSP (ADSP-21161) + fixed point DSP (ADSP-21990) (A/D channels + PWM), Fs=20 kHz.

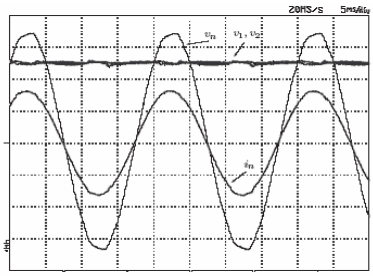
Experimental results II

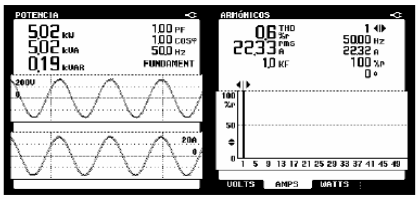




- No load power losses: 150 W, fundamental component of the current in phase with the network voltage.
- Semi-bus DC voltages: balanced.

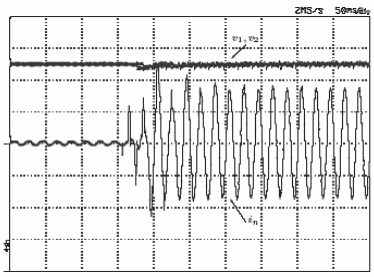
Experimental results III



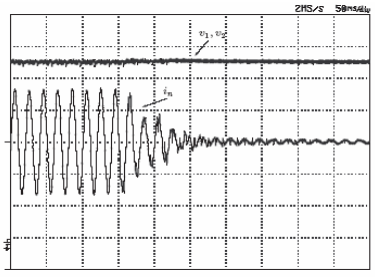


- Network port: PF=1, low current THD (0.6%).
- Semi-bus DC voltages: balanced (in average).

Experimental results IV



No load to full nonlinear load



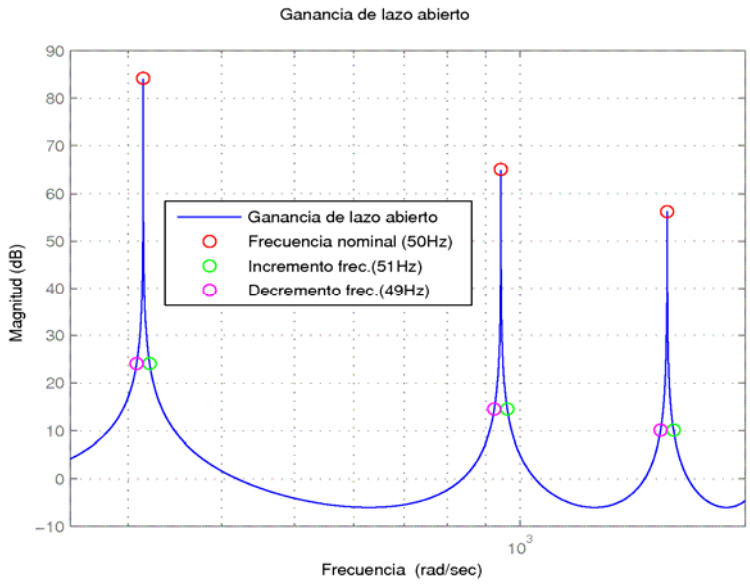
Full nonlinear load to no load

- Steady state reached in about 6 periods (good transient response).
- Semi-bus DC voltages: balanced and low DC bus voltages variation

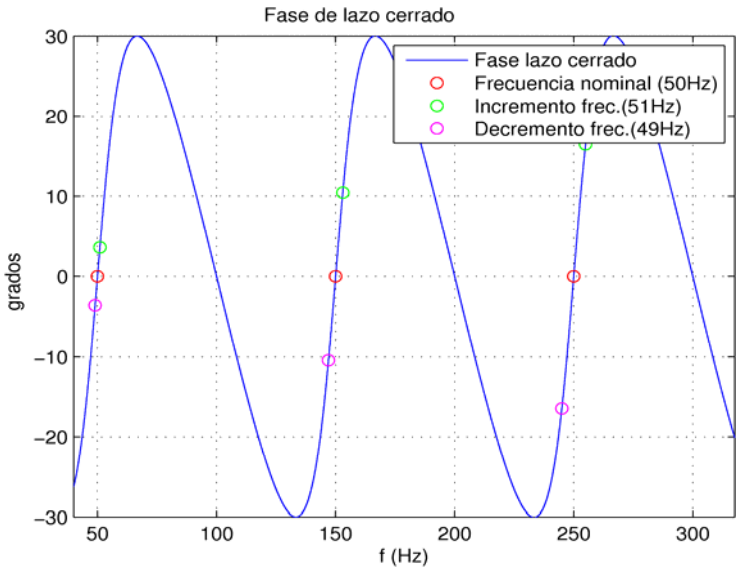
Example

Virtual Laboratory

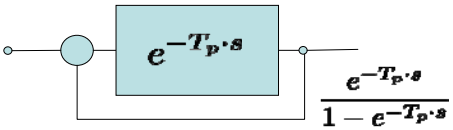
Varying frequency signals (I)



Varying frequency signals (II)



Varying frequency signals (III)



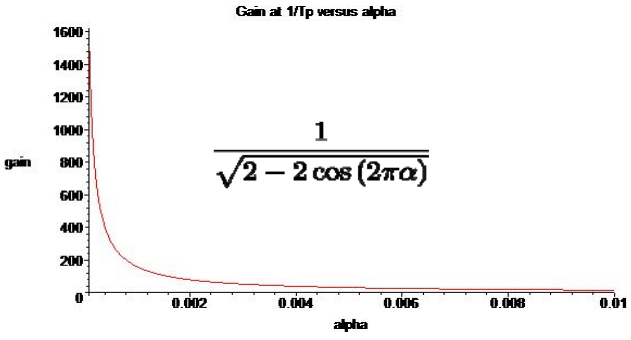
Repetitive Controllers are designed for a certain periodical signal with period T_p

In many circumstance the periodic in uncertain or varies over time.

Multiplicative uncertainty ⟶ $T_p (1+ \prec)$

$$\frac{e^{-T_p \cdot (1+\alpha) \cdot s}}{1 - e^{-T_p \cdot (1+\alpha) \cdot s}}$$

Performance is drastically reduced if the signal frequency is not exactly known



Varying frequency signals (IV)

- Repetitive control for varying frequency signals
  - Variable Sampling Time
  - Constant Sampling Time
    - Higher order repetitive control
    - Fractional delay
    - Interpolation/Delmation

Some References

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- ***Adaptive Feed-Forward Cancellation Control of a Full-Bridge DC-AC Voltage Inverter.*** Shane Malo and Robert Griñó. Proceedings of the 17th World Congress The International Federation of Automatic Control Seoul, Korea, July 6-11, 2008
- ***Digital repetitive plug-in controller for odd-harmonic periodic references and disturbances*** Robert Griñó and Ramon Costa-Castelló. Automatica. Volume 41, Issue 1,Pages 153-157(January 2005)
- ***High Performance Repetitive Control of an Active Filter under Varying Network Frequency.*** Ramon Costa-Castelló, Shane Malo, Robert Griñó. 17th IFAC World congress. July 6-11. Seoul Korea
- ***Odd-Harmonic Digital Repetitive Control of a Single-Phase Current Active Filter.*** Ramon Costa-Castelló, Robert Griñó & Enric Fossas. IEEE Transactions on Power Electronics. Volume: 19, Issue: 4, Year: July 2004. E.Page(s):1060- 1068.