



CONFERENCIA MARTES 29 DE SEPTIEMBRE DE 2026 A LAS 10:00

Input-output differential systems: representations and stability

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Resumen.- We shall consider the following higher order systems of differential equations

$$\mathbf{P}(D)y = \mathbf{Q}(D)u \quad \text{and} \quad u = \tilde{\mathbf{P}}(D)z \ \& \ y = \tilde{\mathbf{Q}}(D)z \quad (1)$$

on the time interval $[0, \infty)$, where D denotes the differential operator and $\mathbf{P}$, $\mathbf{Q}$, $\tilde{\mathbf{P}}$, $\tilde{\mathbf{Q}}$ are polynomial matrices of suitable formats. It is assumed that $\mathbf{P}$ and $\tilde{\mathbf{P}}$ are square, non-singular and the rational matrices $\mathbf{P}^{-1}\mathbf{Q}$ and $\tilde{\mathbf{Q}}\tilde{\mathbf{P}}^{-1}$ are holomorphic in a neighbourhood of infinity. The functions u and y represent input and output signals, respectively, and z is an internal variable, usually referred to as latent variable. To allow for non-smooth u and y , the operator D is interpreted in the sense of distributions. The only regularity constraint on u and y is local integrability.

As the above systems can be written in the form

$$(\mathbf{Q}(D), -\mathbf{P}(D)) \begin{pmatrix} u \\ y \end{pmatrix} = 0 \quad \text{and} \quad \begin{pmatrix} u \\ y \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{P}}(D) \\ \tilde{\mathbf{Q}}(D) \end{pmatrix} z,$$

the first and second system in (1) are referred to as kernel and image representations, respectively.

In the talk, we shall focus on the following three themes.

(1) Analysis of the structure of the spaces

$$\ker (\mathbf{Q}(D), -\mathbf{P}(D)) \cap L^1_{\text{loc}} \quad \text{and} \quad \text{im} \begin{pmatrix} \tilde{\mathbf{P}}(D) \\ \tilde{\mathbf{Q}}(D) \end{pmatrix} \cap L^1_{\text{loc}},$$

where $L^1_{\text{loc}} := \{(u, y) : u \text{ and } y \text{ locally integrable}\}$.

(2) Analysis of the relationship between these two spaces under the assumption that $\tilde{\mathbf{Q}}\tilde{\mathbf{P}} = \mathbf{P}\mathbf{Q}$ or, equivalently, $\mathbf{P}^{-1}\mathbf{Q} = \tilde{\mathbf{Q}}\tilde{\mathbf{P}}^{-1}$.

(3) Stability analysis in the context of suitably generalized concepts of initial-value problems for the systems in (1).

The material will be presented in way which should be understandable for a general mathematical audience. Specialist knowledge of mathematical control and systems theory is not required.

Lugar: Aula Luis Rodríguez Marín del Departamento de Matemática Aplicada de la UNED (Aula 2.32). E.T.S.I. Industriales.