## Departamento de Matemática Aplicada



## CONFERENCIA MARTES 26 DE SEPTIEMBRE DE 2023 A LAS 10:00

## Some Open Problems Involving Big and Little Lip

Bruce Hanson St. Olaf College

Given a continuous function  $f : \mathbb{R} \to \mathbb{R}$  with  $M_f(x,r) = \sup_{|x-y| \le r} |f(x) - f(y)|$ , the so-called "Big Lip" and "Little Lip" functions are defined as follows:

$$\operatorname{Lip} f(x) = \limsup_{r \to 0^+} \frac{M_f(x, r)}{r} \qquad \operatorname{lip} f(x) = \liminf_{r \to 0^+} \frac{M_f(x, r)}{r} .$$

The big Lip function has been around for over one hundred years, while the little lip function is a more recent phenomena, appearing for the first time in the mid 1990's. Although these two functions both measure the "Lipschitzness" of the function f in some sense, they can behave quite differently. For example, it's possible that  $\lim f = 0$  almost everywhere while  $\lim f = \infty$  everywhere! In this talk I will examine this dichotomy and look to characterize the subsets E of the real line which have the property that there exists a function f such that  $\lim f = \infty$  on E and  $\lim f = 0$  on E. I will also pose a number of interesting open problems related to this question.

Lugar: Aula Luis Rodríguez Marín del Departamento de Matemática Aplicada de la UNED (Aula 2.32). E.T.S.I. Industriales.