Departamento de Matemática Aplicada

CONFERENCIA MIÉRCOLES 22 DE FEBRERO DE 2023 A LAS **13:00**

Big and Little lip and quasiconvex spaces

Bruce Hanson
St. Olaf College

Given a metric space (X, d) and $f : X \to \mathbb{R}$ with $D_r f(x) = \sup_{d(x,y)\leq r} \frac{|f(x) - f(y)|}{r}$, the so called "Big Lip" and "Little Lip"functions are defined as follows:

$$\operatorname{Lip} f(x) = \limsup_{r\to 0^+} D_r f(x) \qquad \operatorname{lip} f(x) = \liminf_{r\to 0^+} D_r f(x).$$

Then we define

$$D(X) = \{f : X \to \mathbb{R} \,:\, ||\mathrm{Lip} f(x)||_\infty < \infty\} \;\; d(X) = \{f : X \to \mathbb{R} \,:\, ||\mathrm{lip} f(x)||_\infty < \infty$$

$$\mathrm{Lip}(X) = \{f : X \to \mathbb{R} \,|\, f \text{ is Lipschitz on } X\}.$$

Note that $\mathrm{Lip}(X) \subset D(X) \subset d(X)$.
The metric space (X, d) is called quasiconvex if there exists $K < \infty$ such that given any points $x, y \in X$ there exists a curve γ connecting x and y such that $l(\gamma) \leq K d(x, y)$, where $l(\gamma)$ is the length of the curve γ.
If X is quasiconvex, then $\mathrm{Lip}(X) = D(X) = d(X)$. This turns out to be straightforward to prove. Some interesting questions arise when exploring the converse relations, i.e. if $\mathrm{Lip}(X) = D(X)$ or $D(X) = d(X)$ what can we conclude about the convexity of X? This talk presents joint work in this area by the speaker and Estibalitz Durand Cartagena.

Lugar: Aula Luis Rodríguez Marín del Departamento de Matemática Aplicada de la UNED (Aula 2.32). E.T.S.I. Industriales.