Departamento de Matemática Aplicada

CONFERENCIA LUNES 4 DE JULIO DE 2022 A LAS **12:00**

An introduction to the big and little lip functions

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Given a continuous function $f : \mathbb{R} \to \mathbb{R}$ with $M_f(x, r) = \sup_{|x-y|\leq r} |f(x) - f(y)|$, the so-called "Big Lip" and "Little Lip" functions are defined as follows:

$$\operatorname{Lip} f(x) = \limsup_{r\to 0^+} \frac{M_f(x,r)}{r} \qquad \operatorname{lip} f(x) = \liminf_{r\to 0^+} \frac{M_f(x,r)}{r}.$$

The behavior of these functions is intimately related to the differentiability of f. The Rademacher-Stepanov Theorem tells us that f is differentiable almost everywhere on the set $L_f = \{x : \operatorname{Lip} f(x) < \infty\}$. On the other hand, as Balogh and Csörnyei showed, this theorem no longer holds if we replace L_f with $l_f = \{x : \operatorname{lip} f(x) < \infty\}$. They give an example where $\operatorname{lip} f(x) = 0$ a.e. but $\operatorname{Lip} f(x) = \infty$ for all $x \in \mathbb{R}$ so $L_f = \emptyset$ and f is nowhere differentiable. However, they also show that if $l_f = \mathbb{R}$, then f is differentiable on a set of positive measure and thus L_f has positive measure as well. In this talk, I explore the relationship between $\operatorname{lip} f$ and $\operatorname{Lip} f$ as well as between L_f and l_f. I will also pose a number of open problems.

Lugar: Aula Luis Rodríguez Marín del Departamento de Matemática Aplicada de la UNED (Aula 2.32). E.T.S.I. Industriales.